



## Common Core State Standards Professional Learning Module Series

# Mathematics:

## Kindergarten through Grade Twelve Standards for Mathematical Practice

CALIFORNIA DEPARTMENT OF EDUCATION  
Tom Torlakson, State Superintendent of Public Instruction

---

### Talking Points:

- Welcome to the “Mathematics: Kindergarten through Grade Twelve Standards for Mathematical Practice” module, a part of the Common Core State Standards (CCSS) for California Educators Professional Learning Module Series.
- There are two math modules: This module, which covers mathematical practices, and another titled, “Mathematics: Kindergarten through Grade Eight Learning Progressions” which covers the kindergarten through grade eight content standards for mathematics.
- This module explores the teaching and learning of mathematics through the lens of the Standards for Mathematical Practice for students in kindergarten through grade twelve. It will deepen educators’ understanding of what the mathematical practices are, and why they are important to bring to their students as they transition to the CCSS.

# Welcome to the Series

## Common Core State Standards (CCSS) Professional Learning Modules

Available on the Brokers of Expertise Web Site at [www.myboe.org](http://www.myboe.org)

### Welcome Message

Lupita Alcala , Deputy Superintendent

Instruction and Learning Support Branch

California Department of Education (CDE)

<http://myboe.org/portal/default/Group/Viewer/GroupView?action=2&qid=2996>

2 | California Department of Education

## Optional Slide

---

### Facilitator Notes:

- Refer to the Brokers of Expertise Web site for an overview of CCSS modules available.
- Play the welcome message by Lupita Alcala, Deputy Superintendent of the Instruction and Learning Support Branch at the California Department of Education.

## Introduction and Overview Standards for Mathematical Practice

- Module Goals
- Overview of the CCSS for Mathematics
- Introduction to Module Units
- Metacognition and Reflection
- Pre-Assessment

3 | California Department of Education

---

### Note to Facilitator:

- Remind participants to turn off cell phones or put in silent mode and refrain from texting.
- Review table handouts and supplies.

### Talking Points:

- This first section of the module (Introduction, Overview, and Unit 1) will take approximately two hours to complete.
- In the Introduction and Overview we will cover the following topics:
  - Module Goals
  - Overview of the CCSS for Mathematics
  - Introduction to Units
  - Metacognition and Reflection
  - Pre-Assessment

## Module Goals

The overarching goals of the “Mathematics: Kindergarten through Grade Twelve (K–12) Standards for Mathematical Practice” module are to enable educators to:

- Deepen their understanding of the Common Core Mathematical Practice (MP) standards
- Support the learning of all students

4 | California Department of Education

---

### Facilitator Notes:

Emphasize that the goals of this module are for teachers to understand the mathematical practice standards and to be able to apply this knowledge to support the learning of their students.

- “SMP” refers to the eight practice standards as a group
- “MP” is used to refer to each individual standard

# The CCSS Initiative

Supports states in developing tools to support teaching and learning:

- Curricula
- Instructional materials
- Assessments
- Professional learning opportunities

The CCSS are:

- Internationally benchmarked
- Focused on readiness for college and career

5 | California Department of Education

---

## Talking Points:

- First, let's review some basic information about the CCSS.
- The **Common Core State Standards Initiative** was a state-led effort coordinated by the **National Governors Association Center for Best Practices** and the **Council of Chief State School Officers**.
- The CCSS are designed for the purpose of providing strong, shared expectations across states and allows the adopting states to develop, create, and share high-quality tools to support teaching and learning, including:
  - Curricula
  - Instructional materials
  - Assessments
  - Professional development programs
- The CCSS are:
  - **Internationally benchmarked** with high-performing countries and are designed to prepare students for college and career readiness, allowing American students to successfully compete and collaborate with students around the world.
  - **Focused on readiness for college and career** and provide a clear and consistent framework to prepare students for future academic and career success. They provide a common overview of the knowledge and skills K–12 students need, upon graduation from high school, to succeed in credit-bearing, academic college courses and in workforce training programs.

# The CCSS for Mathematics

## Two Types of Mathematics Standards:

- **Standards for Mathematical Content:** Define what students should understand and be able to do at each grade level.
- **Standards for Mathematical Practice (SMP):** Ensure that the “processes and proficiencies of mathematics” are at the heart of teaching and learning.

These two types of standards are interconnected; interacting to improve instruction and learning for all students.

6 | California Department of Education

---

## Talking Points:

- The CCSS for Mathematics define what students should understand and be able to do. These standards include expectations for both content and practice.
- The **Standards for Mathematical Content**, commonly referred to as “content standards”, include skills and understandings that students should develop at each grade level (or within each course)
- The **Standards for Mathematical Practice**, commonly referred to as “practice standards”, identify the processes and proficiencies that students should develop throughout their entire K–12 mathematics education.

# Introduction to SMP Module Units

- Unit 1: Teaching and Learning the SMP
- Unit 2: Overarching Habits of Mind (MP1 and MP6)
- Unit 3: Reasoning and Explaining (MP2 and MP3)
- Unit 4: Modeling and Using Tools (MP4 and MP5)
- Unit 5: Seeing Structure and Generalizing (MP7 and MP8)
- Unit 6: Summary, Next Steps, and Resources

7 | California Department of Education

---

## Facilitator Note:

Review list of unit titles on slide.

## Talking Points:

- Using the lens of the SMP, this module will guide you through the teaching and learning of mathematics to foster an understanding of the eight mathematical practices and why they are important for your students.
- The rationale for this structure and grouping of the SMP is provided in Unit 1.

# Metacognition and Reflection

**Metacognition:** Thinking about thinking, reflecting on one's personal thought processes, allowing the learner to connect new knowledge to prior knowledge, formulate ideas, and clarify thinking.

- You will have the opportunity to predict, reflect, make conjectures, and synthesize through:
  - entries in your Metacognitive Journal
  - small and large group discussion

8 | California Department of Education

---

## Facilitator Notes:

- Define “metacognition” on slide, indicating that participants will be journaling thoughts in a “Metacognitive Journal” and collaborating to review and discuss reflections together.



# Pre-Assessment

Assess your knowledge of the SMP prior to beginning the lessons:

- Complete the “Pre-Assessment” (**Handout 1.0**)
- Work independently, without discussion or assistance from others

9 | California Department of Education

---

## Facilitator Notes:

- This activity will take approximately 5 minutes.
- Refer participants to the “Pre-Assessment” (**Handout 1.0**) and indicate that they should NOT collaborate on answering the questions.
- Facilitator can decide whether to review correct answers or wait until the end of the workshop to allow participants to learn over time if their responses were correct or not.
- An “I-Clicker” or similar tool may be used for group review with answers provided OR left open-ended, at the discretion of the facilitator. Participants should return the completed handout to their packets for future reference.

## Talking Points:

- Before beginning the module, you will participate in a pre-assessment. This activity will assess your knowledge of the SMP prior to beginning the lessons.
- The assessment will be repeated upon completing the module activities to assess your learning.

## Pre-Assessment answers:

1. c
2. c
3. d
4. a
5. c
6. True
7. False
8. False
9. False
10. True

# Unit 1

## Teaching and Learning the Standards for Mathematical Practice

10 | California Department of Education

---

### Talking Points:

Now let's begin Unit 1: *Teaching and Learning the Standards for Mathematical Practice*.

## Unit 1: Learning Objectives

Upon completion of this unit:

- You will be able to describe the difference between the Standards for Mathematical Content and the SMP.
- You will be introduced to the eight SMP and understand their importance and connection to the content standards.
- You will be able to explain how the SMP define what it means for a student to be mathematically proficient.

11 | California Department of Education

---

### Facilitator Notes:

Review content on slide.

# 1.0 Observing Students

Observe students engaged in mathematics:

**Grade 2: Classroom Discussion: Describing Patterns**

**Grade 6: Classroom Discussion: Sorting Angles**

**Grade 8: Classroom Discussion: Speed and Rate Using Technology**

As you watch the video(s), think about the following:

- What questions are being asked and at what level?
- What strategies are being used?

Videos available on the Brokers of Expertise Web site at

<http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11776>

12 | California Department of Education

---

## Talking Points:

- Observing students engaged in talking about and doing mathematics allows teachers to identify the SMP in action.
- We will watch a video [or three videos] of students engaged in doing mathematics. As you watch, take note of how the students are engaging in mathematics.

## Facilitator Notes:

- Link to video(s) hosted on the Brokers of Expertise Web site:  
<http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11776>
- Videos are less than 4 minutes each.
- Options for viewing:
  - 1) For multilevel groups, view all three videos as a group.
  - 2) For homogenous grade span groups, view appropriate video as a group.
  - 3) Divide participants in to three groups, primary, upper elementary, secondary. Each group will view the video at their grade span (as technology availability allows).

Grade 2: 3 min. 45 sec.

Grade 6: 3 min. 15 sec.

Grade 8: 2 min. 30 sec.

## Observing Students, Cont.

### Discuss in small groups:

- What were the students doing?
- In what ways were they engaged?
- Describe your observations of students' thinking, problem solving, and interactions.

13 | California Department of Education

---

### Facilitator Notes:

- After viewing the video, have participants discuss questions on slide in grade level groups (10 minutes)
- Have each group share out key ideas to the whole group (5 minutes)

## 1.1 Content Standards: Important Shifts

The content standards are organized around domains and coherent clusters. They seek to illustrate increased:

- **Focus:** Strongly emphasized at appropriate grade levels for deep understanding
- **Coherence:** Thinking across grade levels, and linking to major topics within grade levels
- **Rigor:** Pursue, with equal intensity, conceptual understanding; procedural skill and fluency; and application in major topics

14 | California Department of Education

---

### Talking Points:

- To establish a context for the SMP, this section will provide a brief overview of the Standards for Mathematical Content.
- The CCSS focus on fewer topics to allow for a greater in-depth examination of mathematics concepts. As a result of a clearly defined coherence across grade levels, students' conceptual understanding develops to allow for mastery of mathematical concepts.
- The content focuses on a rigorous set of skills at each grade level, enabling students to build a strong foundational understanding of mathematics. For example, in the early grades there is a greater emphasis on number concepts and skills. The key to student understanding is a balance of skill and procedure with a clear focus on conceptual development.
- Standards that are rigorous, coherent, and focused provide better guidance to educators, students, and parents about desired learning outcomes. The standards aim for clarity and specificity, and also stress conceptual understanding of key ideas. In addition, the standards provide a balance of procedural skills and conceptual development to support students' mathematical understanding.

# Content Standards: Balance and Depth of Understanding

## K–8 Content Standards by Domains

K	1	2	3	4	5	6	7	8
Geometry								
Measurement and Data					Statistics and Probability			
Number and Operations in Base Ten					The Number System			
Operations and Algebraic Thinking					Expressions and Equations			
Counting and Cardinality			Number and Operations—Fractions			Ratios and Proportional Relationships	Functions	

- Examples of tasks can be viewed at the standards level in each grade.

Source: Illustrative Mathematics: <http://illustrativemathematics.org>

15 | California Department of Education

### Talking Points:

- The Illustrative Mathematics Web site offers interactive access to the content standards for K–8 by Content Domains, High School Conceptual Categories, and the SMP.
- Note that Geometry is the only Domain that goes across the grades. Counting and Cardinality is only at kindergarten and lays the foundation for numbers. Fractions start in grade three and are tied directly to number concepts. Domains have a beginning and an end, with expectations that students learn the concepts and ideas in depth.
- See progressions documents for further information: <http://commoncoretools.me/category/progressions/>

### Note to Facilitator:

- Copy and paste the Illustrative Mathematics link into a Web browser to navigate the site with participants: <http://illustrativemathematics.org/standards/k8>
- If participants have access to the Internet, have them take few minutes to navigate within the interactive graph on the K–8 section of the Web site.

## Balance and Depth of Understanding, Cont.

### High School Conceptual Categories



- The Modeling Conceptual Category (not represented in the chart) includes standards in Higher Mathematics Courses and is embedded throughout other Conceptual Categories indicated by a (?) symbol.
- Examples of tasks can be viewed at the standards level in the conceptual categories.

Source: Illustrative Mathematics <http://illustrativemathematics.org>

16 | California Department of Education

---

### Talking Points:

- The Web site also shows the following graph which is accessible by Conceptual Category. The Modeling Conceptual Category (not represented in the chart) includes standards in Higher Mathematics Courses and is embedded throughout other Conceptual Categories indicated by a (?) symbol.
- In addition, examples of tasks can be viewed at the standards level.

### Note to Facilitator:

- Copy and paste the Illustrative Mathematics link into a Web browser to navigate the interactive graph on the Illustrative Mathematics Web site with participants: <http://illustrativemathematics.org/standards/hs>
- If participants have access to the Internet, have them take few minutes to navigate on their own.



## Additional Information

- “Mathematics: Kindergarten Through Grade Eight Learning Progressions” professional learning module:
  - Brokers of Expertise <http://myboe.org/>
- Tools for the Common Core Standards:  
<http://commoncoretools.me/category/progressions/>

17 | California Department of Education

---

### Talking Points:

- For additional information about the content standards, visit the mathematics learning progressions online module on the Brokers of Expertise Web site. This module provides an overview of how the CCSS for Mathematics kindergarten through grade eight learning progressions are sequenced across and within grade-level spans (kindergarten through grade five and grades six through eight) and how to apply the progressions to instructional practice.
- Visit also the “Tools for the Common Core Standards” Web site at the address provided on this slide.

## 1.2 The Practice Standards

The mathematical practices “rest on important ‘processes and proficiencies’ with longstanding importance in mathematics education.

Source: CCSS for Mathematics, p.6

21 | California Department of Education

---

### Facilitator Notes:

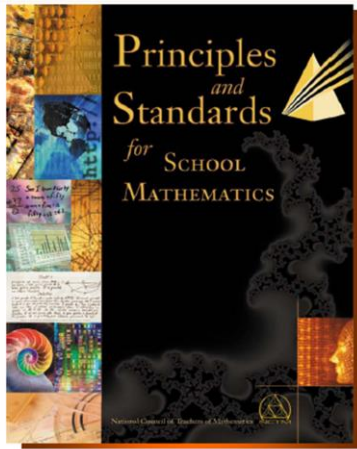
Slides 18–26 give information on the development and foundational underpinnings of the practice standards.

### Handouts for this section:

- Slide 19: Handout 1.2.1: “Principles and Standards for School Mathematics: Process Standards”
- Slide 20: Handout 1.2.2: “Intertwined Strands of Proficiency”
- Slide 23,24: Handout 1.2.3: “The Eight Standards for Mathematical Practice”
- Slide 26: Handout 1.2.4: “Self-Reflection Survey”

## Processes:

### National Council of Teachers of Mathematics (NCTM)



NCTM (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

19 | California Department of Education

- Problem Solving
- Reasoning and Proof
- Communication
- Representation
- Connections

---

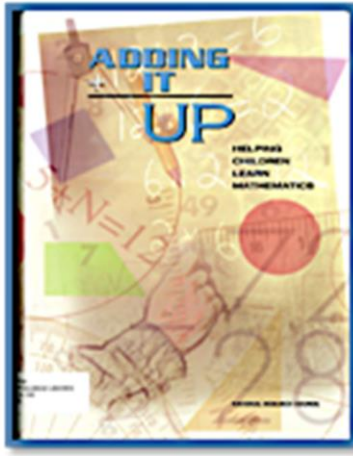
#### Handout (optional):

“Principles and Standards for School Mathematics: Process Standards” (**See Handout 1.2.1**)

#### Talking Points:

- The SMP processes are derived from the National Council of Teachers of Mathematics process standards, outlined in the publication, *Principles and Standards for School Mathematics* (2000).
- There are five process standards for mathematical practice, as listed on this slide.
- Take a few minutes to review Handout 1.1.1

## Proficiencies: National Research Council



- Adaptive Reasoning
- Strategic Competence
- Conceptual Understanding
- Procedural Fluency
- Productive Disposition

23 | California Department of Education

---

### Handout (optional):

“Intertwined Strands of Proficiency” (See Handout 1.2.2)

### Talking Points

- The SMP proficiencies are based on the **strands of mathematical proficiency** specified in the National Research Council’s report, *Adding It Up: Helping Children Learn Mathematics* (2001).
- The processes and proficiencies provided the basis for the development of the SMP; ways of thinking and doing that lead to successful learning of mathematics content.
- Take a few minutes to review the handout titled, “Intertwined Strands of Proficiency”

# Doing and Using Mathematics

The SMP are about doing and using mathematics to foster reasoning and sense-making.



21 | California Department of Education

---

## Talking Points:

- The practice standards are about doing and using mathematics to foster reasoning and sense-making.
- Expertise with these practices emerges over time from opportunities and experiences provided in the mathematics classroom.

## Ongoing Work

- The practice standards are not intended to be a list of processes and proficiencies to be checked off for mastery or completion.
- Student performance may range from novice to expert based on the content, the activities, and the task in which the student is engaged.

22 | California Department of Education

---

### Talking Points:

- Work on the SMP is ongoing.

### Note to Facilitator:

- Review slide



## The Eight SMP

**MP1: Make sense of problems and persevere in solving them**

**MP2: Reason abstractly and quantitatively**

**MP3: Construct viable arguments and critique the reasoning of others**

**MP4: Model with mathematics**

23 | California Department of Education

---

**Handout:** “The Eight Standards for Mathematical Practice” (See Handout 1.2.3)

**Note to Facilitator:**

- Direct participants to handout and review the detail of each standard

**MP1. Make sense of problems and persevere in solving them**

Making sense and persevering are habits of mind needed by all students to be successful learners of mathematics. Before a student can engage in mathematics, they need to make sense of what they are being asked to consider.

**MP2. Reason abstractly and quantitatively**

Reasoning abstractly requires that students make sense of quantities and their relationships in problem situations. Students decontextualize and contextualize mathematics; they translate problem situations into symbols which they are able to manipulate and, as they manipulate the symbols, refer back to the problem situation to make sense of their work.

**MP3. Construct viable arguments and critique the reasoning of others**

Constructing arguments requires that students use stated assumptions, definitions, and previous results. They make conjectures, justify their conclusions, and communicate them to others. They respond to the arguments of others.

**MP4. Model with mathematics**

Modeling with mathematics requires that students make assumptions and approximations to simplify a situation, realizing these may need revision later, and that students interpret mathematical results in the context of the situation and reflect on whether they make sense.

## The Eight SMP, cont.

**MP5. Use appropriate tools strategically**

**MP6. Attend to precision**

**MP7. Look for and make use of structure**

**MP8. Look for and express regularity in repeated reasoning**

24 | California Department of Education

---

**Note to Facilitator:** Continue from previous slide.

### **Talking Points:**

#### **MP5. Use appropriate tools strategically**

Using tools strategically requires that students are familiar with appropriate tools to decide when each tool is helpful, know both benefits and limitations, detect possible errors, and identify relevant external mathematical resources and use them to pose or solve problems.

#### **MP6. Attend to precision**

Precision refers to the accuracy with which students use mathematical language and symbols as well as precision in measurement.

#### **MP7. Look for and make use of structure**

Looking for structure refers to students' understanding and using properties of number systems, geometric features and relationships, and patterns of a variety of types to solve problems.

#### **MP8. Look for and express regularity in repeated reasoning**

Looking for regularity in repeated reasoning refers to the process of noticing repeated patterns or attributes and using those to abstract and express general methods, expressions or equations, or relationships.



## For All Learners, All the Time

- Each practice standard begins with the three words, “**mathematically proficient students.**”
- Proficiency is a goal for ALL students.
- The practice standards are for all students at all times, regardless of their abilities.

25 | California Department of Education

---

### Talking Points:

- The SMP define what it means for students to be mathematically proficient.
- In order for the practice standards to be developed, the approaches should be embedded in all mathematical activities such as instruction, discussion, and investigation; not just during problem-solving sessions such as “Problem of the Day.”
- This does not mean that all eight SMP are in play at all times.
- Engagement with the SMP is not experienced in isolation or for special times within the class period.

## Self-Reflection Survey

How comfortable and confident are you in supporting all students to be successful in the eight mathematical practices?

26 | California Department of Education

---

### Handout:

“Self-Reflection Survey” (See Handout 1.2.4).

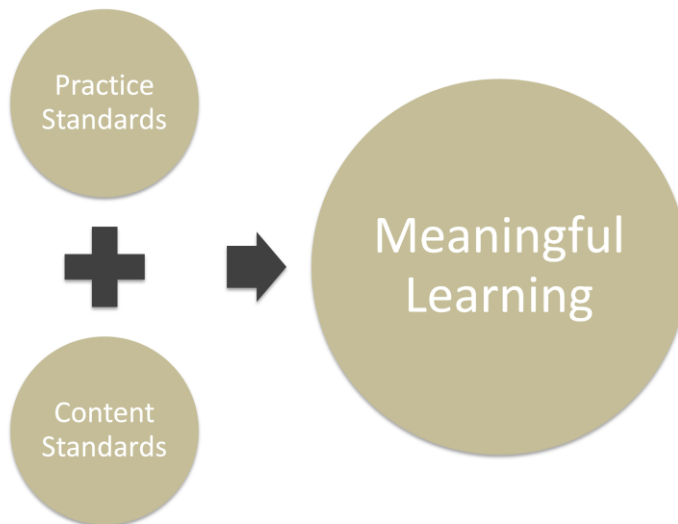
### Note to Facilitator:

- Administer Self Reflection Survey or assign as writing prompt
- Save survey or writing entries for comparison with end of module response

### Talking Points:

- At the end of the module, you will return to this survey to see how your participation in this module has impacted your confidence in teaching and learning the eight SMP.
- The unit continues with a look at connecting content and practice standards, organizing for implementation and meeting the needs of all students.

## 1.3 Interaction of Practice Standards and Content Standards



27 | California Department of Education

---

### Talking Points:

- The next few slides will provide an overview of how to achieve meaningful learning for ALL students through the interaction of both practice and content standards for mathematics.

### Notes to Facilitator:

Section includes:

- Slide 27: Interaction of Practice Standards and Content Standards
- Slide 28: Meaningful Learning
- Slide 29: Focus and Coherence (video)
- Slide 30: Time to Talk
- Slide 31: Bringing Structure
- Slide 32: The Structure

# Meaningful Learning

Meaningful learning requires:

- Access to a coherent and connected curriculum
- An environment that promotes discourse, reflection, collaboration, and use of appropriate tools
- Doing mathematical problems and tasks which necessitate problem solving, reasoning, and sense-making

28 | California Department of Education

---

## Talking Points:

- Meaningful learning is likely to occur when students have access to a coherent and connected curriculum in an environment that promotes discourse, reflection, collaboration, and use of appropriate tools while working on challenging mathematics.
- This involves doing mathematical problems and tasks which necessitate problem solving, reasoning, and sense-making.
- Student engagement with both content and practice is needed for deep learning.

## Focus and Coherence

The video, “Mathematical Practices, Focus and Coherence” provides an overview of the interrelationship of the mathematics content and practice standards.

- Features Jason Zimba, one of the lead writers of the CCSS for Mathematics

Video available on the Council of Chief State School Officers  
Web site at

[http://programs.ccsso.org/ccv/JZ\\_4.m4v](http://programs.ccsso.org/ccv/JZ_4.m4v)

29 | California Department of Education

---

### Note to Facilitator:

- Video length: 1:13 minutes
- Video also available at:  
<http://www.youtube.com/watch?v=9pKcO9E4Flw&list=UUF0pa3nE3aZaFBMT8pgM5PA&index=10&feature=plcp>

### Talking Points:

- The video, “Mathematical Practices, Focus and Coherence,” featuring Jason Zimba (one of the lead writers of the CCSS for Mathematics) underscores the interrelation of the two types of standards, mathematical content and mathematical practice.

## Time to Talk

- What are some of your initial observations about the eight SMP?
- What are some commonalities of the SMP across the grade levels?
- What are some ways in which you might connect content standards with the SMP in your classroom?

30 | California Department of Education

---

### Notes to Facilitator:

Facilitate a discussion based on the questions presented.

- Step 1 Options:
  - Elbow partners
  - Small groups
  - Individual quick write
- Step 2: Full group discussion

## Bringing Structure

“If you think about it long enough you can associate just about any practice standard with any content standard, but this sort of matrix thinking can lead to a dilution of the force of the practice standards — if you try to do everything all the time you end up doing nothing.”

Bill McCallum, March 10, 2011

31 | California Department of Education

---

### Talking Points:

- Many of the processes and proficiencies in the individual practice standards overlap, and several may be used together in solving a problem or engaging in a task.
- Additionally, most of the practices can be applied to most of the content standards. An attempt has been made to bring structure to the SMP.
- In order to avoid “doing nothing” when integrating the practice and content standards, Bill McCallum, one of the lead writers of the *CCSS for Mathematics*, grouped the standards to bring a higher order of structure to the standards.

# The Structure

## CCSS Mathematical Practices

### OVERARCHING HABITS OF MIND

1. Make sense of problems and persevere in solving them
6. Attend to precision

### REASONING AND EXPLAINING

2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others

### MODELING AND USING TOOLS

4. Model with mathematics
5. Use appropriate tools strategically

### SEEING STRUCTURE AND GENERALIZING

7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

32 | California Department of Education

### Talking Points:

- The eight practice standards have been usefully grouped into four pairs which provides structure to the mathematical practices just as the clusters and domains bring structure to the content standards.
- This structure serves as a way to think about and plan for implementation of the SMP as well as to select tasks and look for evidence of students' demonstration of engagement.
- It makes logical connections across the standards and takes advantage of the overlap of practice standards to provide needed focus.
- The structure is shown in the diagram. It has also been used to organize the units within this module.



# Connecting Content and Practice

## Points of Intersection:

- The SMP do not exist in isolation; doing and using mathematics involves connecting content and practice.
- The content standards are based on procedure and on understanding — both are stressed equally.



33 | California Department of Education

## Talking Points:

- The next group of slides provides information on connecting content and practices.

## Notes to Facilitator:

The section includes:

- Slide 33: Connecting Content and Practice
- Slide 34: Impact of Understanding
- Slide 35: Importance of Understanding
- Slide 36: Possible Starting Points
- Slide 37: An Example
- Slide 38: Grain Size
- Slide 39: Organizing for Learning
- Slide 40: Progress Check

Handouts for this section:

- CCSS for Mathematics (**See Handout 1.3.1**).

## Impact of Understanding

“Students who lack understanding of a topic may rely on procedures too heavily.... In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.”

Source: CCSS for Mathematics

34 | California Department of Education

---

### Facilitator Notes:

- Allow time to read slide.

### Talking Points:

- In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice.

## Importance of Understanding

“Without understanding, a student may rely on procedures and may not represent problems coherently, justify conclusions, apply the mathematics to other situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an interview, or deviate from a known procedure to find a shortcut.”

Source: CCSS for Mathematics

35 | California Department of Education

---

### Facilitator Notes:

- Allow time for participants to read silently. May follow up with oral reading.

### Talking Points:

Understanding is important in implementation of the CCSS for mathematics.

## Possible Starting Points

Content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the SMP.

36 | California Department of Education

---

### Talking Points:

A certain level of understanding is needed in order to employ the SMP to deepen learning.

## An Example

### **Grade 4, Number and Operations — Fractions, Standard 3a:**

Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

Source: CCSS for Mathematics

37 | California Department of Education

---

#### **Talking Points:**

- In this standard (Grade 4, Number and Operations—Fractions, Standard 3a), over-reliance on an algorithm may make it impossible for a student to explain, for example, why reference to the same whole is necessary when adding fractions or to build on notions of equivalence or knowledge of unit fractions to justify a solution.
- Therefore, the concepts delineated at these points of intersection are important; as is providing time, resources, and focus for their development. Setting the points of intersection as a priority can only lead to improved teaching and learning of mathematics.

#### **Facilitator Notes:**

Activity follow up:

- Direct participants to locate other content standards which set an expectation of understanding.
- Pose question: What connections can be made between SMP and the content standards for the specific standard chosen?

## Grain Size

“It is not expected that a single problem would engage students with all eight SMP; even a lesson may be too small to accomplish that.”



Source: Daro, 2010

38 | California Department of Education

---

### Talking Points:

- Phil Daro, one of the lead writers of the *CCSS for Mathematics*, refers to proper grain size as the unit at which it makes most sense to organize mathematics for learning.
- Daro suggests that teachers look at units or chapters rather than lessons when planning for instruction and assessment of the standards.

## Organizing Mathematics for Learning

Over the course of such a larger segment of work, engagement of multiple standards should be evident rather than within a single lesson, task, or problem.

39 | California Department of Education

---

### Note to Facilitator:

- Review slide

## Progress Check

So far, this unit has covered:

- Grouping practices to facilitate implementation
- Connecting practice and content standards
- Organizing mathematics

Write/Discuss: How might what you have learned so far impact your instructional practice?

40 | California Department of Education

---

### Facilitator Notes:

- Provide a brief review and direct participants to reflect on the prompt.
- If using a journal, provide time to reflect and write.
- Facilitate a discussion based on the questions presented.
  - Step 1 Options:
    - Elbow partners
    - Small groups
    - Individual quick write
  - Step 2: Full group discussion



## 1.4 Meeting the Needs of ALL Students

The CCSS for Mathematics are for ALL students, including:

- Students receiving special education services
- English learners
- Underperforming students
- Gifted and talented students



41 | California Department of Education

---

**Notes to Facilitator:** Review slide

Slides 41–50 contain information on meeting the needs of all students.

- Slide 41: Meeting the Needs of All Students
- Slide 42: Language Learning and Mathematics
- Slide 43: Students with Disabilities
- Slide 44: Resources
- Slide 45: Supporting English Learners: English Language Development Standards
- Slide 46: Understanding Language
- Slide 47: Teaching Els Conceptually (video link)
- Slides 48-49: 21<sup>st</sup> Century Skills
- Slide 50: Learning Objectives Revisited

### Talking Points:

- The *CCSS for Mathematics* demands that the needs of ALL students are addressed.
- The units of this module will give you the opportunity to think about differentiation strategies to meet the needs of all learners, including students with special needs, English learners, under-performing students, and gifted and talented students.

# Language Learning and Mathematics

- Balance student activities that address both **conceptual** and **procedural** knowledge
- Use high cognitive demand tasks
- Instill beliefs that mathematics is sensible, worthwhile, and doable
- Provide opportunities for students to engage in all of the mathematical practices

Source: Moschkovich, 2012

42 | California Department of Education

---

## Talking Points:

- The SMP emphasize *doing* mathematics. Therefore, all students must have an opportunity to demonstrate learning in an environment that structures timeframes in which to talk, access tools, work on challenging tasks, and other activities supported in the SMP.
- Judit Moschkovich, professor of mathematics education at Stanford University, whose research examines the relationship between language learning and math, recommends ways to support the learning of language in conjunction with the teaching of mathematics. The instructional strategies shown on the slide can be applied for ALL students of mathematics (Moschkovich, 2012).

## Students with Disabilities

The standards are “for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs.” (CCSS for Mathematics).

Accommodations include:

- Instructional supports for learning
- Instructional accommodation
- Assistive technology devices and services

43 | California Department of Education

---

**Facilitator Notes:** Read quote.

### Talking Points:

- Students with disabilities will receive the support they need to *do* mathematics. These accommodations include:
  - **Instructional supports for learning** — foster student engagement by presenting information in multiple ways and allowing for diverse avenues of action and expression
  - **Instructional accommodations** — changes in materials or procedures which do not change the standards but allow students to learn within the framework of the CCSS
  - **Assistive technology devices and services** — ensure access to the general education curriculum and the CCSS

## Students with Disabilities: Resources

- ***Application to Students with Disabilities***  
<http://www.corestandards.org/assets/application-to-students-with-disabilities.pdf>
- ***Teaching Common Core Math Practices to Students with Disabilities***  
<http://www.naset.org/3553.0.html>
- ***The Special EDge***  
[http://www.calstat.org/publications/pdfs/Edge\\_summer\\_2012\\_newsletter.pdf](http://www.calstat.org/publications/pdfs/Edge_summer_2012_newsletter.pdf)
- ***Teaching Mathematics CCSS to Students with Moderate to Severe Disabilities*** <http://cec.metapress.com/content/80146k52rpk56813/>
- ***Six Principles for Principals to Consider in Implementing CCSS for Students with Disabilities***  
<http://www.naesp.org/principal-septemberoctober-2012-common-core/access-common-core-all-0>

44 | California Department of Education

---

### Facilitator Notes:

May choose to distribute “Application to Students with Disabilities” and “Teaching Common Core Math Practices to Students with Disabilities” as a handout.

### Talking Points:

- The CCSS publication, “Application to Students with Disabilities”, addresses how the standards can apply to students with disabilities and their special needs, and ensures access to the general curriculum.
- “Teaching Common Core Math Practices to Students with Disabilities” offers alternatives to direct instruction for special education teachers.
- The “Special EDge” is a publication of the California Department of Education’s Special Education Division. Read the issue devoted to the CCSS.
- The Council for Exceptional Children (CEC) views the CCSS as an opportunity to provide access to the general curriculum to all students with disabilities. The journal article, “Teaching Mathematics CCSS to Students with Moderate to Severe Disabilities”, provides teachers with a six-step approach to providing instruction to students with moderate and severe disability aligned to the new CCSS.
- Another CED publication, “Six Principles for Principals to Consider in Implementing CCSS for Students with Disabilities,” provides CCSS implementation guidance in the instruction and assessment of students receiving special education services.

## Supporting English Learners: English Language Development Standards

*“English Language Development Standards (ELD) for California Public Schools: Kindergarten through Grade Twelve” prioritizes **meaning and interacting** over language structure.*

Language is emphasized as “a complex and social meaning-making resource to be fostered via intellectually challenging, interactive, and dialogue-rich contexts focused on content knowledge and linguistic development.”

ELD Standards Overview, p.18

45 | California Department of Education

---

### Note to Facilitator:

Link for overview of ELD standards: <http://www.cde.ca.gov/sp/el/er/documents/sbeoverviewpld.pdf>

### Talking Points:

- In order to ensure equity in the mathematics classroom, the needs of English learners must be considered. The *CCSS for Mathematics* speaks to teaching mathematics for understanding. Part of this understanding is embedded in language proficiency for mathematics.
- California has revised and updated its *English Language Development Standards for California Public Schools: Kindergarten through Grade Twelve*. [Review slide quote]
- HANDOUT: *Overview of the California English Language Development Standards and Proficiency Level Descriptors*
- OPTIONAL HANDOUT: “*Mathematics, the Common Core, and Language: Recommendations for Mathematics Instruction for ELs Aligned with the Common Core*” by Judit Moschkovich (**See Handout 1.4.2**).

Available at <http://ell.stanford.edu/sites/default/files/pdf/academic-papers/02-JMoschkovich%20Math%20FINAL.pdf>

## Understanding Language: Guiding Principles

1. Focus on students' mathematical reasoning, not accuracy, in using language.
2. Shift to a focus on mathematical discourse practices; move away from simplified views of language.
3. Recognize and support students to engage with the complexity of language in mathematics classrooms.
4. Treat everyday language and experiences as resources, not as obstacles.
5. Uncover the mathematics in what students say and do.

46 | California Department of Education

---

### Talking Points:

- Understanding Language is a initiative at Stanford University that focuses on heightening awareness of the language and literacy issues embedded within the new standards. Judit Moschkovich is part of the leadership team for this initiative. Her publication, *Mathematics, the Common Core, and Language: Recommendations for Mathematics Instruction for ELs Aligned with the Common Core*, provides principles to guide educators in supporting reasoning and sense-making of mathematics for English learners. [[http://ell.stanford.edu/sites/default/files/pdf/academic-papers/02\\_JMoschkovich%20Math%20FINAL.pdf](http://ell.stanford.edu/sites/default/files/pdf/academic-papers/02_JMoschkovich%20Math%20FINAL.pdf)]

### Guiding Principles:

- **Focus on students' mathematical reasoning, not accuracy, in using language.** Focus instruction on uncovering, hearing, and supporting students' mathematical reasoning, not on accuracy of language.
- **Shift to a focus on mathematical discourse practices; move away from simplified views of language.** The focus of classroom activity should be on student participation in mathematical discourse practices (explaining, conjecturing, justifying, etc.). Instruction should move away from simplified views of language as words, phrases, vocabulary, or a list of definitions.
- **Recognize and support students to engage with the complexity of language in mathematics classrooms..** Language in mathematics classrooms is complex and involves:
  - Multiple modes (oral, written, receptive, expressive, etc.).
  - Multiple representations (including objects, pictures, words, symbols, tables, graphs, etc.)
  - Different types of written texts (textbooks, word problems, student explanations, teacher explanations, etc.)
  - Different types of talk (exploratory and expository)
  - Different audiences (presentations to the teacher, to peers, by the teacher, by peers, etc.)
- **Treat everyday language and experiences as resources, not as obstacles.** It is not useful to dichotomize academic and everyday language. Instead, instruction needs to consider how to support students in connecting the two ways of communicating, building on everyday communication, and contrasting the two when necessary.
- **Uncover the mathematics in what students say and do.** Materials and professional development should support teachers to prepare them to deal with the tensions around language and mathematical content, in particular: How to uncover the mathematics in student contributions; When to move from everyday to more mathematical ways of communicating; When and how to approach and develop "mathematical precision."

Using the above principles as guides, the standards may be implemented in ways that all students achieve and are equipped to meet the challenges of learning mathematics.

## Teaching Mathematics for Conceptual Understanding for ELs



<http://ell.stanford.edu/publication/mathematics-common-core-and-language>

47 | California Department of Education

---

### Note to Facilitator:

Video available on the Understanding Language Web site at:

- <http://ell.stanford.edu/publication/mathematics-common-core-and-language>
- <http://www.youtube.com/watch?v=gUfpnlbg4TA>

### Talking Points:

- Let's take a few minutes to watch Moschkovich's video on teaching mathematics for conceptual understanding with English learners (3:49 minutes).
- ELs may not speak like mathematicians right away, but teachers can build on the mathematical reasoning they're already doing.
- As an extension activity, you may read the entire paper by Judit Moschkovich, "*Mathematics, the Common Core, and Language: Recommendations for Mathematics Instruction for ELs Aligned with the Common Core*" or her classroom vignette based on a lesson presented in a third grade bilingual classroom in California.



# 21<sup>st</sup> Century Skills

Students must have an opportunity to develop the skills, processes, and knowledge that will prepare them for the challenges that they will face. 21st Century Skills include:

- **Creativity and innovation**
- **Critical thinking and problem solving**
- **Collaboration**
- **Communication**
- **Construction and exploration of new understandings**

Education Code Section 60207(d)

48 | California Department of Education

---

## Talking Points:

- At the heart of the *CCSS for Mathematics* is a mandate to prepare ALL students for college, career, and life. To do so effectively, students must have an opportunity to develop the skills, processes, and knowledge that will prepare them for the challenges that they will face. These skills are known as 21<sup>st</sup> Century skills.

- (1) **Creativity and innovation**—including, but not limited to thinking creatively, working creatively with others, and implementing innovations
- (2) **Critical thinking and problem solving**—including, but not limited to, reasoning effectively, using systems thinking, making judgments and decisions, and solving problems
- (3) **Collaboration**—including, but not limited to, working effectively in diverse teams, adapting to change and being flexible, demonstrating initiative and self-direction, working independently, demonstrating productivity and accountability, and demonstrating leadership and responsibility
- (4) **Communication**—including, but not limited to, communicating clearly and effectively through reading, writing, and speaking.
- (5) **Construction and exploration of new understandings**—knowledge through the integration of content from one subject area to another and multiple modes for demonstrating innovative learning



# 21<sup>st</sup> Century Skills and the SMP

An example of interconnectedness (Source: P21 Common Core Toolkit)

21 <sup>st</sup> Century Skill	Standards for Mathematical Practice
Critical thinking and problem solving	<ul style="list-style-type: none"><li>• Make sense of problems and persevere in solving them</li></ul>
	<ul style="list-style-type: none"><li>• Reason abstractly and quantitatively</li></ul>
	<ul style="list-style-type: none"><li>• Model with mathematics</li></ul>
	<ul style="list-style-type: none"><li>• Look for and make use of structure</li></ul>

49 | California Department of Education

---

## Talking Points:

- 21<sup>st</sup> Century skills are strongly related to the SMP. The Partnership for 21<sup>st</sup> Century Skills (P21) provides examples of this in its publication, *P21 Common Core Toolkit: A Guide to Aligning the Common Core State Standards with the Framework for 21st Century Skills* (p. 8).
- Engagement in the SMP allows for the development of 21<sup>st</sup> century skills, as shown in this slide.

P21 Web site: [http://p21.org/index.php?option=com\\_content&task=view&id=1005&Itemid=236](http://p21.org/index.php?option=com_content&task=view&id=1005&Itemid=236)

## Learning Objectives Revisited

- Describe the difference between the Standards for Mathematical Content and the SMP.
- Understand the importance of the eight standards for mathematical practice.
- Explain how the standards for mathematical practice define what it means for a student to be mathematically proficient.

50 | California Department of Education

---

### Talking Points:

- Let's revisit our learning objectives. At this point you should be able to...

## Talk about...

- Some of the new ideas you have gained in this introductory unit
- Questions you have
- Some of the challenges that you anticipate as you prepare to support your students in their use of the SMP

51 | California Department of Education

---

### Facilitator Note:

- Facilitate a discussion regarding the questions or assign a written reflection followed by sharing.

### Talking Points:

- By this time, you should be able to do the following: [Review slide]

### Facilitator Note:

Slides 51–53 close Unit 1 and allow time for reflection and/or discussion

- Slide 51: Talk about ...
- Slide 52: Think about
- Slide 53: Looking Ahead

## Think about...

**As you participate in the next four units, think about:**

- How will you teach so that students may engage in the SMP?
- How will you assess the SMP?
- How will you support students as they deepen their mathematical understanding through engagement in the SMP?
- How will you provide opportunities for **all** students to engage in the SMP?

52 | California Department of Education

## Looking Ahead

**As you work through Units 2–5, consider:**

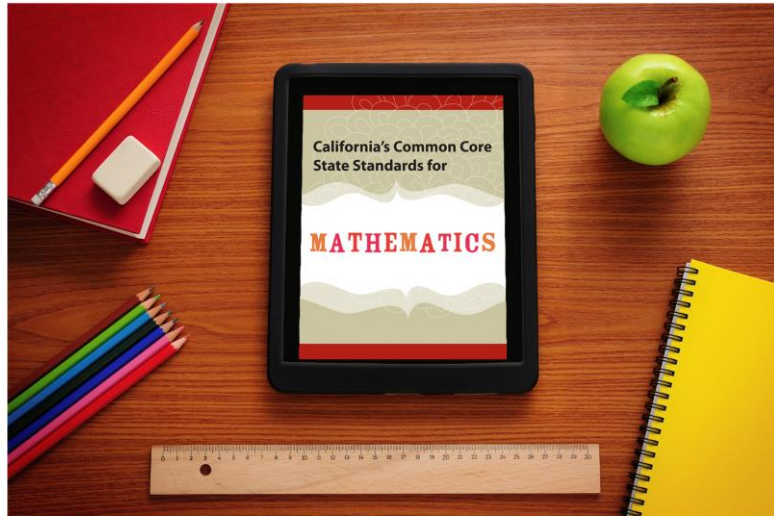
- Trying out some of the tasks in your classroom
- Sharing the ideas learned with your colleagues

**Now onward to a greater understanding of the Standards for Mathematical Practice!**

53 | California Department of Education

- 
- Unit 2: Overarching Habits of Mind (MP 1 and MP 6)
  - Unit 3: Reasoning and Explaining (MP 2 and MP 3)
  - Unit 4: Modeling and Using Tools (MP 4 and MP 5)
  - Unit 5: Seeing Structure and Generalizing (MP 7 and MP 8)
  - Unit 6. Summary, Next Steps, and Resources

# California's Common Core State Standards for Mathematics



0 minutes (TRANSITION SLIDE)

---