Consecutive Sums

Some numbers can be written as a sum of consecutive positive integers:

$$6 = 1 + 2 + 3$$
$$15 = 4 + 5 + 6$$
$$= 1 + 2 + 3 + 4 + 5$$

Which numbers have this property? Explain.

Let's look at what might be expected of students at each grade span when working on this problem.

K–2 Example

Write the first 10 numbers as a sum of other numbers. Which of these sums contain only consecutive numbers?

3–5 Example

Some numbers can be written as a sum of consecutive positive integers, for example:

$$\begin{aligned} 6 &= 1 + 2 + 3 \\ 15 &= 4 + 5 + 6 \\ &= 1 + 2 + 3 + 4 + 5 \end{aligned}$$

In small groups, find all numbers from 1–100 that can be written as a consecutive sum. Look for patterns as you work. Conjecture which numbers can and which cannot be expressed as a consecutive sum. How can some of the sums be used to find others?

6–8 Example

Some numbers can be written as a sum of consecutive positive integers, for example:

$$\begin{aligned} 6 &= 1 + 2 + 3 \\ 15 &= 4 + 5 + 6 \\ &= 1 + 2 + 3 + 4 + 5 \end{aligned}$$

Which numbers have this property?

Sally made this conjecture: "Powers of 2 cannot be expressed as a

consecutive sum."

Agree or disagree and explain your reasoning.

9–12 Example

Some numbers can be written as a sum of consecutive positive integers, for example:

$$6 = 1 + 2 + 3$$
$$15 = 4 + 5 + 6$$
$$= 1 + 2 + 3 + 4 + 5$$

Exactly which numbers have this property?

When investigating this problem, Joe made the following conjecture: "A number with an odd factor can be written as a consecutive sum and the odd factor will be the same as the number of terms." Agree or disagree with this statement and explain your reasoning.