

Kindergarten through Grade 12 Standards for Mathematical Practice

User's Guide



California Department of Education
Sacramento, California

K–12 Standards for Mathematical Practice
User’s Guide

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Additional Resources	Refer to online module (unit 6)

Group Facilitator’s Guide: Preparation Notes

Background Information for Facilitator(s)

- Complete the *Kindergarten through Grade Twelve Standards for Mathematical Practice (K–12 SMP)* online module; available on the Brokers of Expertise Web site at <http://ccssplm.myboe.org>
- Review the outline and scope of the companion online module, *Mathematics: Kindergarten through Grade Eight Learning Progressions*; also available on the Brokers of Expertise Web site at <http://ccssplm.myboe.org>
- Be familiar with the text of each Mathematical Practice (MP) standard
- Be familiar with the following CCSS Web sites:
 - Inside Mathematics: <http://insidemathematics.org/>
 - Illustrative Mathematics: <http://www.illustrativemathematics.org/>
- Be familiar with the Understanding Language Web site: <http://ell.stanford.edu/>
- Keep in mind that the overarching goals of the K–12 SMP module is two-fold:
 - Deepen participants’ understanding of the SMP
 - Support the learning of all students
- The K–12 SMP module has six units:
 - Units 1 and 6 are the introduction and summary
 - Units 2–5 each focus on two of the eight SMP

Group Facilitator’s Guide: Preparation Notes

Materials, Supplies, and Equipment

Print or provide electronic copies in advance (located at <http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11908>):

- **PowerPoint Slides with Presenter Notes** (pdf of PowerPoint in “notes view”) —copy for facilitator use only
- **Participant packets**—one per person (print only the units you plan to cover each session, or have participants download documents onto personal electronic devices in advance)
 - Participant Packet (pdf) — Presentation Slides
 - Participant Packet (pdf) — Handouts

Note: Most handouts have three numbers. The first number refers to the unit, the second refers to the section in that unit (which could be the zero section), and the third is the order in which they appear in that section. For example, Handout 2.1.2 is from Unit 2, Section 1, and is the second handout in that section.

Introduction and Overview Handout:

- Handout 1.0: Pre-Assessment

Unit 1 Handouts:

- Handout 1.2.1: Principles and Standards for School Mathematics Process Standards (NCTM Process Standards)
- Handout 1.2.2: Intertwined Strands of Proficiency (Adding It Up)
- Handout 1.2.3: The Eight Standards for Mathematical Practice
- Handout 1.2.4: Self-Reflection Survey
- Additional Unit 1 Resources: Links to articles (for download and print)
- Optional Handout (all Units): Recommended resource to have on hand

Unit 2 Handouts:

- Handout 2.0: Compare Standards MP1 and MP6 (Venn diagram)
- Handout 2.0.1: Standards for Mathematical Practice MP 1 and MP 6
- Handout 2.1.1: Sierpinski’s Triangle
- Handout 2.1.2: Teacher Reflections
- Handout 2.1.3: Posing Questions – Primary
- Handout 2.1.4: Posing Questions – Upper Elementary
- Handout 2.1.5: Posing Questions – Middle School
- Handout 2.1.6: Posing Questions – High School
- Handout 2.2.1: The Hook to Persevere – Primary Task
- Handout 2.2.2: The Hook to Persevere – Upper Elementary Task

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- Handout 2.2.3: The Hook to Persevere – Middle School Task
- Handout 2.2.4: The Hook to Persevere – High School Task
- Handout 2.3.1: Focusing on Mathematical Statements

Unit 3 Handouts:

- Handout 3.0.1: Standards for Mathematical Practice MP2 and MP3
- Handout 3.0.2: Comparing Standards
- Handout 3.1.1: Even Survey
- Handout 3.1.2: Even Survey: Student Responses
- Handout 3.1.3: Sample Strategies for Differentiating Instruction
- Handout 3.1.4: Lesson Plan for Day One
- Handout 3.2.1: Taxonomy of Questions: Questions and Responses
- Handout 3.2.2: Level 3: Noticing Patterns Across Multiple Examples
- Handout 3.2.3: Engaging Diverse Learners
- Handout 3.3.1: Identifying Flaws Through Discourse
- Handout 3.4.1: MARS Task: Flower Arranging
- Handout 3.4.2: Brandon’s Flower Arrangement
- Handout 3.4.3: Partners Sharing
- Handout 3.4.4: David, Josue, and Michael’s Poster
- Handout 3.4.5: Student-Generated Math Challenge

Unit 4 Handouts:

- Handout 4.0.1: Standards for Mathematical Practice MP4 and MP5
- Handout 4.1.1: Tiling Pool Problem (all grades)
- Handout 4.2.1: Quotes on Modeling
- Handout 4.2.2: Examples of Grades K–2 Modeling Activities
- Handout 4.2.3: Examples of Grades 3–5 Modeling Activities
- Handout 4.2.4: Examples of Grades 6–8 Modeling Activities
- Handout 4.2.5: Examples of Grades 9–12 Modeling Activities
- Handout 4.3.1: Quotes on Tools
- Handout 4.4.1: Using Tools: K–2 Task and 3–5 Task
- Handout 4.4.2: Using Tools: Middle School
- Handout 4.4.3: Using Tools: High School

Unit 5 Handouts:

- Handout 5.0.1: MP7 and MP8
- Handout 5.1.1: Consecutive Sums
- Handout 5.2.1: Square Tiles
- Handout 5.3.1: K–2 Geometry Example
- Handout 5.3.2: Grades 6–8 and 9–12 Geometry Examples
- Handout 5.4.1: K–2 Example: Building Walls
- Handout 5.4.2: Grades 3–5 Example: Hexagons in a Row
- Handout 5.4.3: 6–8 Example: Necklaces
- Handout 5.4.4: 9–12 Example: Sidewalk Patterns

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Unit 6 Handouts:

- Handout 6.1.1: Self-Reflection Survey
- Handout 6.1.2: Post-Assessment

Prepare in Advance:

For each session, have the following available:

- Table supplies (e.g., pens, pencils, highlighters, self-stick notes)
- Chart paper and markers (optional)
- Computer (with Flash player capability-test in advance)
- LCD projector
- Document camera
- Speakers for video
- Internet connection (access to YouTube, Vimeo, etc., if applicable)
- Notebook or notepad for each participant (for journal entries)

Materials to Prepare by Unit:

Note: Videos are accessed through links provided on PowerPoint slides, or within the online module at

<http://myboe.org/portal/default/Content/Viewer/Content?action=2&scId=306591&scIId=11908>.

Introduction and Overview:

- Welcome video by Lupita Alcala

Unit 1:

- Five Unit 1 videos

Unit 2:

- Three Unit 2 videos
- Centimeter rulers
- Coordinate graph paper
- Protractor and straight edge

Unit 3:

- Ten Unit 3 videos
- Multi-colored cubes

Unit 4:

- Blue and White Color Tiles (or other tiles with two colors)
- Coordinate graph paper
- Calculators
- Colored pencils
- Five Unit 4 videos

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Unit 5 Materials:

- Seven Unit 5 videos and one optional video

Technology considerations:

- Verify internet connection
- Verify access to YouTube, Vimeo, etc., if needed
- Verify access to embedded videos (requires Flash)

Presentation Options

Facilitators may conduct a series of group workshops for the entire module, or facilitate a hybrid model where group learning is blended with independent learning. If the hybrid model is chosen, the facilitator should determine the appropriate areas for group facilitation and for independent use. For instance, many sections require participants to do a mathematics problem in a specific grade span. Facilitators could ask participants to complete some tasks independently at their specific grade span to prepare for discussion and reflection in grade span groups. This may work particularly well with a K–12 audience.

Following are presentation options (in addition to the Introduction and Overview):

- Option 1: Three four-hour sessions
 - Session One: Units 1–2
 - Session Two: Units 3–4
 - Session Three: Units 5–6

NOTE: Facilitator can assign tasks for participants to do independently between sessions. Some units are lengthy and may require additional time to complete if some tasks are not done independently.

- Option 2: Two six-hour sessions
 - Session One: Units 1–3
 - Session Two: Units 4–6
- Option 3: Six two-hour sessions
 - Session One: Unit 1
 - Session Two: Unit 2
 - Session Three: Unit 3
 - Session Four: Unit 4
 - Session Five: Unit 5
 - Session Six: Unit 6

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- Option 4: Four one-hour sessions with online independent learning (hybrid; combination of synchronous and asynchronous learning)
 - Session One Facilitated: One-hour Webinar to provide an overview of the K–12 SMP module and Units 1 and 2.
 - Session One Independent: Participants complete Units 1 and 2 online asynchronously.
 - Session Two Facilitated: One-hour Webinar to respond to questions about Units 1 and 2 and provide an overview of Units 3 and 4.
 - Session Two Independent: Participants complete Units 3 and 4 online asynchronously.
 - Session Three Facilitated: One-hour Webinar to respond to questions about Units 3 and 4 and provide an overview of Units 5 and 6.
 - Session Three Independent: Participants complete Units 5 and 6 online asynchronously.
 - Session Four Facilitated: One-hour Webinar to respond to questions about Units 5 and 6 and provide a summary of K–12 SMP Module.

Note: If the audience is a specific grade span, the facilitator can focus only on the tasks and activities for that grade span.

Optional Activities

- Provide additional time for participants to work on mathematics problems.
- Provide additional time to highlight the conceptual understanding needed for participants to understand the mathematics content.
- Provide additional time for participants to reflect on the tasks—the problems, the videos, and the student work—for them to gain a depth of understanding of the nature of the mathematics.
- Provide additional time for participants to explore the problems on the Inside Mathematics and Illustrative Mathematics Web sites.
- Provide additional time for participants to read recommended articles.

Group Facilitator’s Guide: Section Notes

Introduction and Overview: K–12 Standards for Mathematical Practice

Slides: 1–9
Suggested Time: 30 minutes
<p>This section includes:</p> <ul style="list-style-type: none">▪ Overview of the CCSS for Mathematics▪ Introduction to Module Units▪ Metacognition and Reflection▪ Pre-Assessment <p>Preparation Notes:</p> <ul style="list-style-type: none">▪ Be familiar with the module goals.▪ Make available Handout 1.0: Pre-Assessment for each participant▪ Be familiar with the rationale for using metacognitive journals in the “Thinking about Thinking” section. An alternative to the metacognitive journal is to have the participants reflect and respond to questions orally in small groups. <p>Presentation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ Emphasize that the goal of this module is for participants to understand the SMP and to be able to apply this knowledge to support the learning of all students.

Group Facilitator’s Guide: Section Notes

Unit 1. Teaching and Learning the Standards for Mathematical Practice

Section: 1.0	Observing Students
Slides: 10–13	
Suggested Time: 15 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Learning objectives for Unit 1▪ Videos identifying the SMP in action <p>Preparation Notes:</p> <ul style="list-style-type: none">▪ Be familiar with the Learning Objectives for Unit 1▪ Preview videos to resolve technical issues and become familiar with content. Videos are available on the Brokers of Expertise Web site at http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11776 <p>Presentation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ The videos presented here will be shown again at the end of the module. Participants are viewing the videos to consider their prior understanding of the SMP.	

Group Facilitator’s Guide: Section Notes

Unit 1. Teaching and Learning the Standards for Mathematical Practice

Sections: 1.1–1.2	Content Standards and Practice Standards
Slides: 14–26	
Suggested Time: 30 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Standards for Mathematical Content▪ Balance and depth of understanding of the mathematics content▪ The Standards for Mathematical Practice▪ SMP Processes and Proficiencies▪ Doing and Using Mathematics▪ The Eight SMP▪ For All Learners, All the Time▪ Self-Reflection Survey <p>Preparation Notes:</p> <ul style="list-style-type: none">▪ Be familiar with the processes and proficiencies of the SMP (the processes are derived from NCTM’s <i>Principles and Standards for School Mathematics</i> and the proficiencies from the National Research Council’s report <i>Adding It Up: Helping Children Learn Mathematics</i>).▪ Make available Handouts 1.2.1 (Processes), 1.2.2 (Proficiencies), 1.2.3 (The Eight SMP), and 1.2.4 (Pre-Self Reflection) for each participant. <p>Presentation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.	

Group Facilitator’s Guide: Section Notes

Unit 1. Teaching and Learning the Standards for Mathematical Practice

Section: 1.3	Interaction of Practice Standards and Content Standards
Slides: 27–40	
Suggested Time: 30 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Meaningful learning▪ Mathematical practices—Focus and Coherence (video)▪ Bringing structure▪ Connecting content and practice▪ Impact and importance of understanding (example standard)▪ Grain size (looking at units or chapters rather than lessons when planning for instruction and assessment of the standards)▪ Organizing mathematics for learning▪ Progress check <p>Preparation Notes:</p> <ul style="list-style-type: none">▪ Be familiar with the video on slide 29 where Jason Zimba underscores the interrelation between the two types of standards.▪ Preview the video to resolve any technical issues. Video available on the Council of Chief State School Officers Web site at http://programs.ccsso.org/ccv/JZ_4.m4v <p>Presentation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ Highlight the importance of understanding (see quote on slide 35)	

Group Facilitator’s Guide: Section Notes

Unit 1. Teaching and Learning the Standards for Mathematical Practice

Sections: 1.4 and 1.5	Meeting the Needs of All Students Looking Ahead
Slides: 41–53	
Suggested Time: 45 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Language learning and mathematics▪ Supporting students with disabilities▪ Supporting English learners▪ 21st Century skills and the SMP▪ Learning objectives revisited▪ Looking ahead <p>Preparation Notes:</p> <ul style="list-style-type: none">▪ Be familiar with the article, “Mathematics, the Common Core, and Language: Recommendations for Mathematics Instruction for ELs Aligned with the Common Core” by Judit Moschkovich (available at http://ell.stanford.edu/sites/default/files/pdf/academic-papers/02-JMoschkovich%20Math%20FINAL.pdf).▪ Be familiar with Understanding Language (http://ell.stanford.edu)▪ Be familiar with the article, “Teaching Common Core Math Practices to Students with Disabilities” by Michelle Stephan and Jennifer Smith (available at http://www.naset.org/3553.0.html). <p>Presentation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ Show the video on Teaching Math for Conceptual Understanding with English Learners (3:49 min): Available at http://ell.stanford.edu/publication/mathematics-common-core-and-language▪ Have participants read the articles by Moschkovich and Stephan & Smith▪ Encourage participants to try some of the tasks presented with their students.	

Group Facilitator’s Guide: Section Notes

Unit 2. Overarching Habits of Mind: MP1 and MP6

Section: 2.0	Unpacking MP1 and MP6
Slides: 1–7	
Suggested Time: 15 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Learning objectives for Unit 2▪ Unpacking MP1 and MP6 <p>Preparation Notes:</p> <ul style="list-style-type: none">▪ Be familiar with the Objectives for Unit 2.▪ Be familiar with the text of MP1 and MP6.▪ Make available Handout 2.0 (MP1 and MP6) and Handout 2.0.1 (Venn Diagrams) for each participant. <p>Presentation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ Have participants read MP1 and MP6, highlighting key words or phrases and jotting down their questions about these two SMP.▪ Have participants complete, then compare, their Venn Diagrams, using the questions in Slide 7.▪ Ask participants to journal what they gained from reading MP1 and MP6.	

Group Facilitator’s Guide: Section Notes

Unit 2. Overarching Habits of Mind: MP1 and MP6

Section: 2.1	Sense Making and Mindsets
Slides 8–19	
Suggested Time: 30 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ A mathematics problem▪ Teacher mindsets▪ Workshop reflections▪ Engaging students: Questioning and responding▪ The importance of mindsets <p>Preparation Notes:</p> <ul style="list-style-type: none">▪ Work on the mathematics problem. Anticipate challenges for participants.▪ Make available Handouts 2.1.1 through 2.1.6.▪ Be familiar with Mindsets and the importance of a growth mindset in students and teachers.▪ Be familiar with the video (Slide 16) on questioning and preview to resolve any technical issues. Video available at http://vimeo.com/10774338.▪ Be prepared to discuss the types of teacher responses to student solutions (Slide 16 and Handouts 2.1.3 – 2.1.6). <p>Presentation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ Have participants work on the mathematics problem. Encourage them to persevere and make sense of the problem. Ask questions to push their thinking. Ask why questions. Discuss questions on Slide 9.▪ Encourage participants to discuss other types of responses to student solutions (Slide 15).▪ Slide 18 focus on Mindsets. Encourage participants to think of ways to facilitate a growth mindset in students.	

Group Facilitator’s Guide: Section Notes

Unit 2. Overarching Habits of Mind: MP1 and MP6

Section: 2.2	Student Self-Efficacy and Perseverance
Slides 19–24	
Suggested Time: 30 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Students’ self-efficacy▪ The “hook” to persevere: Set of grade span tasks that provide the “hook”▪ Task summary and reflection <p>Preparation Notes:</p> <ul style="list-style-type: none">▪ Work all of the problems in advance and determine why these problems are engaging to students.▪ Make available the appropriate grade level problems: Handouts 2.2.1 – 2.2.4 for each participant. <p>Presentation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ Have participants work on the problem in their grade span. Ask them why these problems might be engaging to students. Ask questions to push their thinking. Ask why questions.▪ Optional: Have participants prepare a task as instructed on Slide 23.▪ Discuss questions on slide 24.	

Group Facilitator’s Guide: Section Notes

Unit 2. Overarching Habits of Mind: MP1 and MP6

Section: 2.3	Attend to Precision
Slides 25–38	
Suggested Time: 30 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Example of precision▪ Focusing on mathematical statements▪ Analyses of teachers’ statements▪ Examples of students attending to precision▪ Reflection using TIPS (Think, Ink, Pair, Share) <p>Preparation Notes:</p> <ul style="list-style-type: none">▪ Be familiar with students’ statements.▪ Be familiar with what a precise statement means.▪ Make available Handout 2.3.1 for each participant.▪ Be familiar with the examples in Slides 28–32.▪ Preview videos to be familiar the content and to resolve any technical issues. Videos available on the Brokers of Expertise Web site at http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11739 <p>Presentation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ Ensure participants have a working understanding of necessary and sufficient conditions.▪ Go over the statement of a cat and discuss its imperfections. Ask participants for other examples.▪ Have participants read the examples of teacher statements and discuss the analyses of the teachers’ statements.▪ Analyze the questions in the Reflection Guidelines section (Slide 37). At what grade level is a less precise definition acceptable in order to ensure student understanding of a concept? Is precision determined by the vocabulary available to a student at a given grade level? What questions might you ask your students, particularly English learners and students with disabilities, to push their thinking so that they provide the most precise definition possible at their grade level?	

Group Facilitator’s Guide: Section Notes

Unit 2. Overarching Habits of Mind: MP1 and MP6

Section: 2.4	Summary and Reflection
Slides 39–41	
Suggested Time:	15 minutes
<p>This section includes:</p> <ul style="list-style-type: none">▪ Revisiting the learning objectives▪ Final reflection▪ References <p>Preparation Notes:</p> <ul style="list-style-type: none">▪ Review the references <p>Presentation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ Have participants reflect in their metacognitive journal.▪ Select references for participants if they want to go deeper into MP1 and MP6.	

Group Facilitator’s Guide: Section Notes

Unit 3. Reasoning and Explaining: MP2 and MP3

Section: 3.0	Introduction to SMP 2 and SMP 3
Slides: 1–10	
Suggested Time: 30 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Objectives and Overview for Unit 3▪ Unpacking MP2 and MP3▪ Small Group Discussion▪ Introduction to fifth grade classroom—the case for these practices▪ Connections across the standards (ELA and mathematics) <p>Preparation Notes:</p> <ul style="list-style-type: none">▪ Be familiar with the Objectives for Unit 3.▪ Be familiar with the text of MP2 and MP3.▪ Be familiar with the description of the 5th grade classroom. The students in this classroom appear throughout Unit 3. Note that concepts learned apply to all grade level spans, not only to 5th grade.▪ Preview the video to resolve any technical issues and become familiar with content. Video available on the Brokers of Expertise Web site at http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11837.▪ Be familiar with the connections across standards (ELA and mathematics) and the 21st Century skills.▪ Make available Handout 3.0.1 (Standards for Mathematical Practice: MP2 and MP3) for each participant. <p>Presentation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.	

Group Facilitator’s Guide: Section Notes

Unit 3. Reasoning and Explaining: MP2 and MP3

Section: 3.1	Beginning to Reason: Definitions and Conjectures
Slides: 11–23	
Suggested Time: 30 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Difference between mathematical argument and proof▪ Definitions and Conjectures▪ Student definitions and explanations▪ Developing a community of reasoners▪ Meeting the needs of diverse learners▪ Lesson on introduction to conjectures▪ Introducing viable arguments▪ Forming conjectures and achieving consensus <p>Preparation Notes:</p> <ul style="list-style-type: none">▪ Be familiar with the definition of an even number.▪ Preview the videos to resolve any technical issues and become familiar with the content. Videos available on the Brokers of Expertise Web site at: http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11843 http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11844 <p>Presentation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ Ask the participants to discuss the various argument strategies students used in the definition of an even number.	

Group Facilitator’s Guide: Section Notes

Unit 3. Reasoning and Explaining: MP2 and MP3

Section: 3.2	Explaining and Justifying
Slides: 24–44	
Suggested Time: 30 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Explaining and justifying▪ Levels of explanations: Level 1 (abdicated reasoning to an external authority), Level 2 (systematically working through multiple examples), and Level 3 (beginning to notice patterns or trends)▪ Levels of explanations for ALL students <p>Preparation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ Be familiar with the different levels of explanations and examples depicting the different levels.▪ Preview the videos to resolve any technical issues and become familiar with the content. Videos available on the Brokers of Expertise Web site at: http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11845 http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11846 http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11847 http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11848▪ Be familiar with how to provide access to all students. <p>Presentation Options:</p> <ul style="list-style-type: none">▪ In lieu of metacognitive journal entries, have participants discuss the reflection questions in small groups and have the groups report out.	

Group Facilitator’s Guide: Section Notes

Unit 3. Reasoning and Explaining: MP2 and MP3

Section: 3.3	Flaws in Reasoning
Slides: 33–38	
Suggested Time: 30 minutes.	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Common flaws with odds and evens▪ Identifying flaws through discourse▪ Language of explanations and justifications▪ Definition of “rule” <p>Preparation Notes:</p> <ul style="list-style-type: none">▪ Preview the videos to resolve any technical issues and become familiar with the content. Videos available on the Brokers of Expertise Web site at: http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11849 http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11850 <p>Presentation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.	

Group Facilitator’s Guide: Section Notes

Unit 3. Reasoning and Explaining: MP2 and MP3

Section: 3.4	Marking Arguments More Viable
Slides: 54–55	
Suggested Time: 45 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Public definitions of odd and even▪ Revision sequence▪ MARS Task: Flower Arrangements▪ Student solutions and revisions to Flower Arrangements▪ Using objects to share solutions with whole class and revise▪ Group collaboration▪ Student generated math challenge▪ Students describing viable arguments <p>Preparation Notes:</p> <ul style="list-style-type: none">▪ Be familiar with the sequence of students’ work (seven steps)▪ Preview the video to resolve any technical issues and become familiar with the content. Video available on the Brokers of Expertise Web site at: http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11852 <p>Presentation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”. <p>Presentation Options:</p> <ul style="list-style-type: none">▪ In lieu of some of the metacognitive journal entries, have participants discuss the reflection questions in small groups, and have the groups report out.	

Group Facilitator’s Guide: Section Notes

Unit 3. Reasoning and Explaining: MP2 and MP3

Section: 3.5	Summary and Reflections
Slides: 56–61	
Suggested Time: 15 minutes	
This section includes: <ul style="list-style-type: none">▪ Summary of Unit 3▪ Assumptions and claims▪ Final reflections for Unit 3	
Presentation Notes <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.	

Group Facilitator’s Guide: Section Notes

Unit 4. Modeling and Using Tools: MP4 and MP5

Section: 4.0	Introduction to MP4 and MP5
Slides 1–6	
Suggested Time: 30 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Learning objectives and overview for Unit 4▪ Unpacking MP4 and MP5▪ Small group discussion <p>Preparation Notes:</p> <ul style="list-style-type: none">▪ Be familiar with the Objectives for Unit 4.▪ Be familiar with the text of MP4 and MP5▪ Make available Handout 4.0.1 MP4 and MP5 for each participant. <p>Presentation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ Have participants read MP4 and MP5, highlighting key words or phrases that seem particularly cogent to them or that puzzle or intrigue them. Have them jot down questions that have about these two SMP.▪ Have the participants discuss what they wrote down. Refer to the questions on Slide 6.▪ Ask participants to write in their journals what they gained from reading MP4 and MP5.	

Group Facilitator’s Guide: Section Notes

Unit 4. Modeling and Using Tools: MP4 and MP5

Section: 4.1	Introduction to Modeling with Mathematics
Slides: 7–14	
Suggested Time: 30 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Introduction to modeling with mathematics▪ The Program for International Student Assessment (PISA) rubric▪ Examples of modeling with mathematics by grade span▪ Meeting the needs of all learners▪ Section summary and reflection <p>Preparation Notes:</p> <ul style="list-style-type: none">▪ Be familiar with the PISA rubric that defines modeling problems on three different levels.▪ Work out all of the problems in this section and be familiar with possible challenges that participants may have.▪ Make available Handouts 4.1.1 (Tiling Pool, all grades), one per participant.▪ Have color tiles available for elementary participants (blue and white tiles, if possible).▪ Provide graph paper (optional). <p>Presentation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ Participants unfamiliar with modeling with mathematics may believe that some of the problems are lacking information. It may be up to the participant to design parameters or to pose reasonable parameters in order to solve the problem. They need to justify the parameters they use in their solution.	

Group Facilitator’s Guide: Section Notes

Unit 4. Modeling and Using Tools: MP4 and MP5

Section: 4.2	Modeling with Mathematics at Different Grade Level Spans
Slides: 15–21	
Suggested Time: 30 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Examples of modeling with mathematics by grade span▪ Video of students making sense of a problem <p>Preparation notes:</p> <ul style="list-style-type: none">▪ Make available Handouts 4.2.1 (Quotes), 4.2.2 (K–2 Examples), 4.2.3 (3–5 Examples), 4.2.4 (Middle School Examples, and 4.2.5 (High School Examples).▪ Be familiar with all of the examples in the different grade level spans.▪ When selecting examples for participants to work on, include all the needed materials.▪ Preview the video to resolve any technical issues and to be familiar with the content. Video available on the Brokers of Expertise Web site: http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11855 <p>Presentation notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ Modeling with mathematics, particularly level 3 problems, is not well defined and problems often have a variety of acceptable solutions. It is important to allow participants to explore a variety of solutions as long as the parameters of the mathematics are consistent with the problem.	

Group Facilitator’s Guide: Section Notes

Unit 4. Modeling and Using Tools: MP4 and MP5

Section: 4.3	Introduction to Using Appropriate Tools
Slides: 22–32	
Suggested Time: 30 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Tools for each grade span▪ Differentiating instruction using of technology and tools▪ Summary and reflection <p>Preparation notes:</p> <ul style="list-style-type: none">▪ Be familiar with the tools for each grade span.▪ Make available Handout 4.3.1 for each participant.▪ Preview the videos to resolve any technical issues and to be familiar with the content. Videos available on the Brokers of Expertise Web site: https://www.teachingchannel.org/videos/differentiating-in-math?resume=0 https://www.teachingchannel.org/videos/technology-and-geometry <p>Presentation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.	

Group Facilitator’s Guide: Section Notes

Unit 4. Modeling and Using Tools: MP4 and MP5

Section: 4.4	Use of Tools at Different Grade Spans
Slides: 33–38	
Suggested Time: 30 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Examples of tools in the grade spans▪ Video examples of tools▪ Discussion and written reflection▪ Summary of section <p>Preparation notes:</p> <ul style="list-style-type: none">▪ Make available Handout 4.4.1, 4.4.2, and 4.4.3 on Using Tools in the grade spans.▪ Preview the videos to resolve any technical issues and to be familiar with the content. Videos available on the Brokers of Expertise Web site: https://www.teachingchannel.org/videos/teaching-fractions https://www.teachingchannel.org/videos/teaching-subtracting-integers <p>Presentation Notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.	

Group Facilitator’s Guide: Section Notes

Unit 4. Modeling and Using Tools: MP4 and MP5

Section: 4.5	Unit Summary and Reflections
Slides: 39–40	
Suggested Time: 15 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Summary of Unit 4▪ Review of Unit 4 objectives▪ Final reflections for Unit 3 <p>Presentation notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ It is important to end this unit with a two-part reflection: reflection of what participants gained in this unit and reflection on how they will design a task that they can use in their classroom.	

Group Facilitator’s Guide: Section Notes

Unit 5 Seeing Structure and Generalizing (MP7 and MP8)

Section: 5.0	Introduction to MP7 and MP8
Slides: 1–6	
Suggested Time: 30 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Learning objectives and overview for Unit 5▪ Unpacking MP7 and MP8▪ Small group discussion <p>Preparation Notes:</p> <ul style="list-style-type: none">▪ Be familiar with the Objectives for Unit 5.▪ Be familiar with the text of MP7 and MP8.▪ Make available Handout 5.0.1: MP7 and MP8 for each participant. <p>Presentation notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ Have participants read MP7 and MP8, highlighting key words or phrases that seem particularly cogent to them or that puzzle or intrigue them. Have them jot down questions that have about these two SMP.▪ Have the participants discuss their thoughts.	

Group Facilitator’s Guide: Section Notes

Unit 5. Seeing Structure and Generalizing: MP7 and MP8

Section: 5.1	Seeing Structure and Using Repeated Reasoning and Generalization
Slides: 7–16	
Suggested Time: 30 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Consecutive Sums problems across the grade spans▪ Mathematical Structure in Number Systems▪ Generalization▪ Extending generalizations activity (optional) <p>Preparation notes:</p> <ul style="list-style-type: none">▪ Be familiar with the Consecutive Sums problem in all grade spans.▪ Make available Handout 5.1.1 (Consecutive Sums) for each participant.▪ Obtain the publication containing the optional video if you plan to show it (see options below). <p>Presentation notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”. <p>Optional activity:</p> <ul style="list-style-type: none">▪ Obtain the video of Susie (2nd grade student justifying $a + b - b = a$. (See Carpenter, Thomas, P., Franke, Megan Loef, & Levi, Linda. <i>Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School</i>. Portsmouth, NH: Heinemann, 2003).	

Group Facilitator’s Guide: Section Notes

Unit 5. Seeing Structure and Generalizing: MP7 and MP8

Section: 5.2	Making Sense of a Growing Pattern
Slides: 17–21	
Suggested Time:	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Square Tiles problem▪ Student making sense of Square Tiles problem over time▪ Summary and reflection of section 5.2 <p>Preparation notes:</p> <ul style="list-style-type: none">▪ Be familiar with the Square Tiles problem, anticipating challenges that participants might have.▪ Make available Handout 5.2.1: Square Tiles for each participant.▪ Preview the videos to resolve any technical issues and become familiar with content. Both videos available on the Brokers of Expertise Web site: http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11862 <p>Presentation notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ In lieu of writing a metacognitive journal entry, participants can discuss the questions in grade level groups and report out to the whole group.	

Group Facilitator’s Guide: Section Notes

Unit 5. Seeing Structure and Generalizing: MP7 and MP8

Section: 5.3	Structure and Generalization in Geometry
Slides: 22–24	
Suggested Time: 30 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Geometric examples in four grade spans▪ Reflection on examples <p>Preparation notes:</p> <ul style="list-style-type: none">▪ Be familiar with each of the geometric examples and work them out.▪ Make available Handouts 5.3.1 (K–2 and 3–5 Geometry Examples), and 5.3.2 (Middle School and High School Geometry Examples).▪ Video is required for the 6–8 and 9–12 segments. Preview the videos to resolve any technical issues and become familiar with the content. Videos available at: http://timssvideo.com/67 http://illuminations.nctm.org/ActivityDetail.aspx?id=30 <p>Presentation notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.	

Group Facilitator’s Guide: Section Notes

Unit 5. Seeing Structure and Generalizing: MP7 and MP8

Section: 5.4	Performance Tasks and Student Work
Slides: 25–29	
Suggested Time: 30 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Performance tasks in grade spans; determining patterns and generalizing▪ Student work for the tasks▪ Making connections to writing standards▪ Differentiating Instruction with accompanying videos▪ Summary and reflection <p>Preparation notes:</p> <ul style="list-style-type: none">▪ Be familiar with all of the performance tasks, anticipating challenges and questions that participants might have.▪ Make available Handouts 5.4.1 through 5.4.4, (include tasks and student work by grade span).▪ When discussing differentiating instruction, ask participants to share some of their strategies that have been effective in their classrooms. <p>Presentation notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.	

Group Facilitator’s Guide: Section Notes

Unit 5. Seeing Structure and Generalizing: MP7 and MP8

Section: 5.5	Unit Summary
Slides:30–35	
Suggested Time: 15 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Video examples (by grade spans) of using structure or repeated reasoning▪ Review of Unit 5 objectives▪ Reflection <p>Preparation notes:</p> <ul style="list-style-type: none">▪ Preview the videos to resolve any technical issues and become familiar with content. Videos available at: <p>http://insidemathematics.org/index.php/classroom-video-visits/public-lesson-number-operations/178-multiplication-a-divison-problem-3-part-a?</p> <p>http://insidemathematics.org/index.php/classroom-video-visits/public-lessons-numerical-patterning/219-numerical-patterning-introduction-part-a</p> <p>http://insidemathematics.org/index.php/classroom-video-visits/public-lessons-properties-of-quadrilaterals/297-properties-of-quadrilaterals-tuesday-introduction-part-a</p> <p>Presentation notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ Have participants discuss the questions after reviewing the videos.▪ Summarize the unit.	

Group Facilitator’s Guide: Section Notes

Unit 6. Summary, Next Steps, and Resources

Section: 6	Summary of Module
Slides: 1–17	
Suggested Time: 45 minutes	
<p>This section includes:</p> <ul style="list-style-type: none">▪ Module summary▪ Suggested next steps▪ Self-reflection survey▪ Post-assessment▪ Resources and glossary <p>Preparation notes:</p> <ul style="list-style-type: none">▪ Preview the videos to resolve any technical issues and become familiar with content. Videos available on the Brokers of Expertise Web site at http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11776.▪ Make available Handout 6.1.1 (Self-Reflection Survey) and Handout 6.1.2 (Pre-Assessment) for each participant.▪ Be familiar with the resources available. <p>Presentation notes:</p> <ul style="list-style-type: none">▪ Refer to notes section of PowerPoint presentation for “Talking Points”.▪ Distribute certificates of completion if applicable.	

Independent User’s Guide

Navigation Guide

Welcome to the K–12 Standards for Mathematics Practice (SMP) module. This module is designed so that you can complete it in approximately six hours.

You are encouraged to become thoroughly engaged and go above and beyond what is expected by taking some of the problems presented into your classroom(s) and sharing what you learn with other colleagues. Encourage colleagues to go through the module with you and after completing each Unit, discuss what you have learned. Strive to be actively engaged throughout the whole process.

Below are the titles of the six units in the module and the anticipated time to complete each:

- (30 min.) Unit 1: Introduction and Overview, Teaching and Learning the SMP
- (75 min.) Unit 2: Overarching Habits of Mind, MP1 and MP6
- (75 min.) Unit 3: Reasoning and Explaining, MP2 and MP3
- (75 min.) Unit 4: Modeling and Using Tools, MP4 and MP5
- (75 min.) Unit 5: Seeing Structure and Generalizing, MP7 and MP8
- (30 min.) Unit 6: Summary and Next Steps

Ideally, go through each of these units in the order presented. You may complete some units more rapidly, particularly if you are somewhat familiar with the SMP featured in the unit. Some units may take longer if you want to spend more time reflecting or reviewing. It is recommended that you focus on one grade span throughout. If you decide to view, read, or try all grade spans, this module will take longer.

Each unit is self-navigating. Once you begin with section 1 of each of the units, you will be directed throughout the unit.

Materials/Considerations for independent users:

- Computer with Flash player capability
- Speakers
- Internet Connection
- Paper and pencil

Additional Resources

Extensive lists of additional resources are included in Unit 6 of the module.