ROLLER COASTER CALCULATIONS

GRADE LEVEL(S) 8-12

LESSON OBJECTIVE

Students will use and apply algebraic and scientific concepts. They will solve multi-step problems using physical science principles.

BACKGROUND/PRIOR KNOWLEDGE NEEDED

- Evaluate and solve equations for rate and potential and kinetic energy
- Quadratic functions lessons—California Math Council (cmc.org) and National Council (nctm)
- National Science Foundation links

EDUCATION STANDARDS

CA Algebra I:
5.0 Students solve multistep problems.
15.0 Students apply algebraic techniques to solve rate problems

CA Physical Science 8th grade:
1.b. Students know that average speed is the total distance traveled divided by the total time elapsed and that the speed of an object along the path traveled can vary.
1.c. Students know how to solve problems involving distance, time, and average speed.

Ca Physics:
2.a. Students know how to calculate kinetic energy by using the formula $E = \frac{1}{2}mv^2$
2.b. Students know how to calculate changes in gravitational potential energy near Earth by using the formula (change in potential energy) = mgh (h is the change in the elevation)

MATERIALS NEEDED

West Coaster Data Collection and Analysis Sheets

Stopwatches (or iphone with stopwatch)

MOTIVATION

Students visit the pier and collect data and make estimates related to the Roller Coaster.

GROUP/INDEPENDENT WORK

At the Pier:
Students will be put in teams of 3 to 4 students.
The class will visit the pier to collect the data needed for their class's follow up analysis/problem solving.

Back at School:
Prior to follow up using the Analysis sheet, d = rt is practiced by way of flying paper airplanes:
Each team of four makes ONE paper airplane for the team.
Each member flies it once with distance (cm) and time (sec) recorded. The rate for each flight is calculated. In this way, students are given the opportunity to practice manipulating d = rt.
The team calculates the Average of the 4 distances and time and then the Average Rate.
Using the team's average rate, team asked to calculate how long it would take the paper airplane to travel 1000 meters (1km/0.6 mi) if it maintained that rate of speed.

Follow up to data collection and d = rt activity
Teams work on West Coaster Analysis Questions.

Use Jigsaw to peer correct. Each student in the group letters off A, B, C, and D. They move into expert groups for their particular letter. The A's correct #1 and #2; the B's #3 and #4; the C's #5 and #6; the D's #7 and #8. The "experts" then return to their original group and share the solutions to the Analysis Questions and to recap and review solutions/brainstorm for #9 and #10.

MODIFICATION(S)

Simplify and use an activity with simpler/smaller numbers such as the airplane activity.
Amount of independence for analysis sheet depends on algebra skills of the class.

ASSESSMENT/WRAP UP

Students explain in writing the differences in calculating constant speed, d = rt, and that of a roller coaster. They can articulate the factors influencing the roller coaster's speed and time of travel.

The West Coaster

Data Collection

1. Sketch a picture of the roller coaster track below.

2. Estimate the maximum height of the roller coaster in feet.

3. Time how many seconds it takes the roller coaster to go up the first hill.

4. Time the entire length of the ride in seconds.

5. Now go on the ride yourself. Where on the ride are you traveling the fastest? Indicate this point in your sketch.

Distance = Rate x Time
d = rt

You know d and t, how do you calculate r? ______________

You know d and r, how do you calculate t? ______________

Make a paper airplane for your team.
Give each team a letter: A, B, C or D.
Each team member flies the plane once and records the data.

Team member	**Distance (cm)**	**Time (sec)**	**Rate (speed)**
A			
B			
C			
D			
Average for the team	D (avg) =	T (avg) =	R (avg) =

* How many centimeters are in a meter? __________

* How long would it take your team's paper airplane to travel 1000 meter (1 km/a little more than half a mile)? Use your team data averages and assume the plan maintains its rate of speed from start to finish.

Answer Key
The West Coaster Data Collection

1. Sketch a picture of the roller coaster track below.
   Point out that the first drop isn't very far.

2. Estimate the maximum height of the roller coaster in feet.
   Correct answer is 55 feet.

3. Time how many seconds it takes the roller coaster to go up the first hill.
   Will vary - about 6 seconds.

4. Time the entire length of the ride in seconds.
   Will vary - approximately 45 seconds.

5. Now go on the ride yourself. Where on the ride are you traveling the fastest? Indicate this in your sketch.
   Going fastest when the roller coaster reaches its lowest point.

West Coaster Analysis Questions

The following equations for potential energy and kinetic energy will be used in answering the questions below:

PE = mgh
m = mass in kilograms (kg)
g = acceleration of gravity = 9.8 m/s^2
h = height in meters (m)

KE = ½ mv^2
m = mass in kilograms (kg)
v = velocity (m/s)

The unit of both types of energy is joules (J).
Assume the mass of the roller coaster when fully loaded with riders is 1000 kg.

1. What is the height of the first hill in meters? (1 foot = 0.305 meters)

2. What is the potential energy (PE) of the roller coaster at the top of the first hill?

3. If all of this potential energy turned into kinetic energy (KE) during the ride, what is the velocity of the ride? Express your answers in m/s.

4. The Pier claims that the West Coaster reaches speeds of 35 miles per hour. Does this seem consistent with your results from the previous question?
(1 m/s = 2.2 miles per hour)

5. Estimate how long it would take the roller coaster to travel 60 meters.

6. Estimate how long it would take the roller coaster to travel 10 miles. (Use the speed you calculated in #4, not the speed the Pier claims.)

7. Using the time it took the roller coaster to travel the entire length of the track, estimate the length of the track in meters.

8. In the previous question, is our estimate likely to be high or low? Why?

9. Determine the power output of the roller coaster. This can be found by dividing the potential energy the roller coaster has at the top of the first hill by the time it takes the roller coaster to reach the top. The unit for power is watts (the same as a light bulb).

10. Brainstorm two ways the track could be modified so that the roller coaster could travel at a faster speed.

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The unit of both types of energy is joules (J).

Assume the mass of the roller coaster when fully loaded with riders is 1000 kg.

1. What is the height of the first hill in meters? (1 foot = 0.305 meters)

   55 feet x 0.305 m = 16.8 m

2. What is the potential energy (PE) of the roller coaster at the top of the first hill?

   PE = (1000)(9.8)(16.8) = 164,640 J

3. If all of this potential energy turned into kinetic energy during the ride, what is the velocity of the ride? Express your answers in m/s.

   164,640 = ½ $(1000)v^2$ v = 18.1 m/s

4. The Pier claims that the West Coaster reaches speeds of 35 miles per hour. Does this seem consistent with your results from the previous question? (1 m/s = 2.2 miles per hour)

   18.1 m/s x 2.2 mph = 39.8 miles per hour

   Yes, this does seem consistent, especially considering the fact that the friction the roller coaster encounters on the track will not allow all of the potential energy to convert to kinetic energy.

5. Estimate how long it would take the roller coaster to travel 60 meters.

   Estimating 18.1 m/s to be 20 m/s, it would take the roller coaster approx 3 seconds.

6. Estimate how long it would take the roller coaster to travel 10 miles. (Use the speed you calculated in #4, not the speed the Pier claims.)

   At ~40 mph, the roller coaster can travel 20 miles every half hour, so 10 miles will take 15 minutes

7. Using the time it took the roller coaster to travel the entire length of the track, estimate the length of the track in meters.

Distance = ~20 m/s (45 seconds) = 900 m
(Answers will vary if times are different.)

8. In the previous question, is our estimate likely to be high or low? Why?

High for two reasons. We rounded our speed up to 20 m/s. Also, the roller coaster wasn't going that fast the entire time.

9. Determine the power output of the roller coaster. This can be found by dividing the potential energy the roller coaster has at the top of the first hill by the time it takes the roller coaster to reach the top. The unit for power is watts (the same as a light bulb).

164,640 / 6 seconds = 27,440 W
(Answers will vary if times are different.)

10. Brainstorm two ways the track could be modified so that the roller coaster could travel at a faster speed.

- Make the first hill higher so more potential energy could turn into kinetic energy.
- Reduce the friction on the track by applying a lubricant.
- Have the first hill drop lower to the ground so that the potential energy converts to kinetic energy sooner.