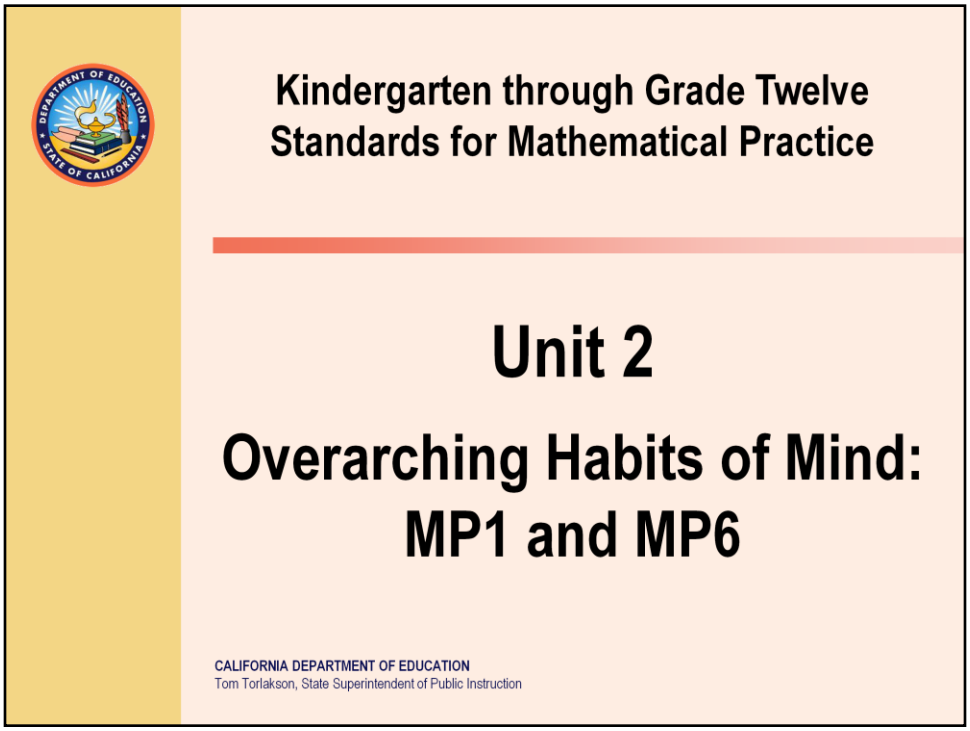
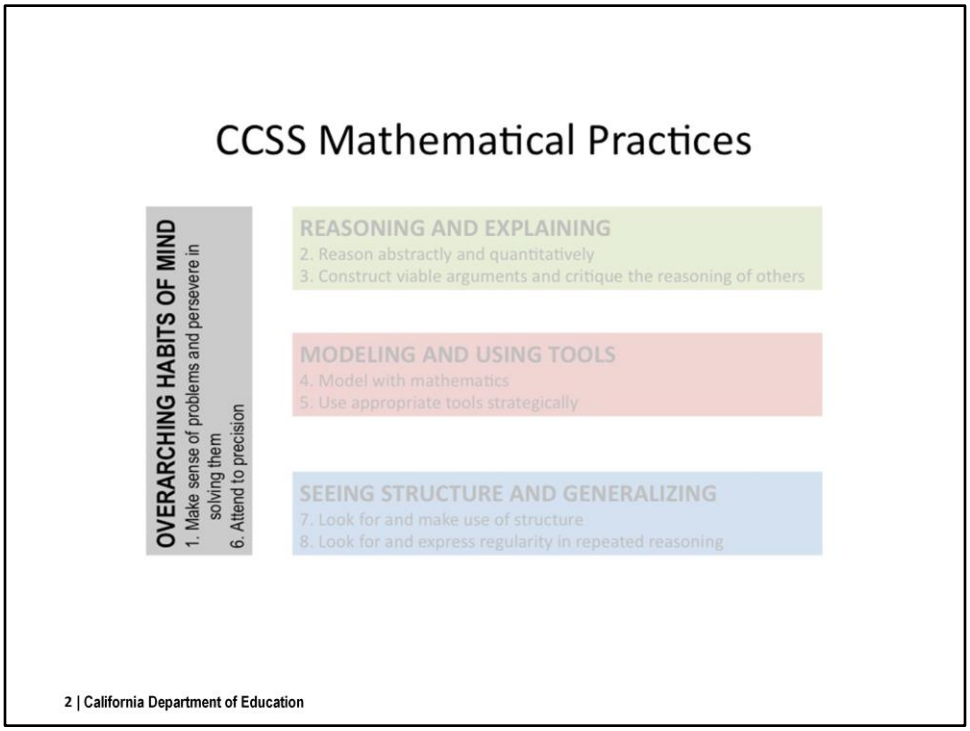


Talking Points:

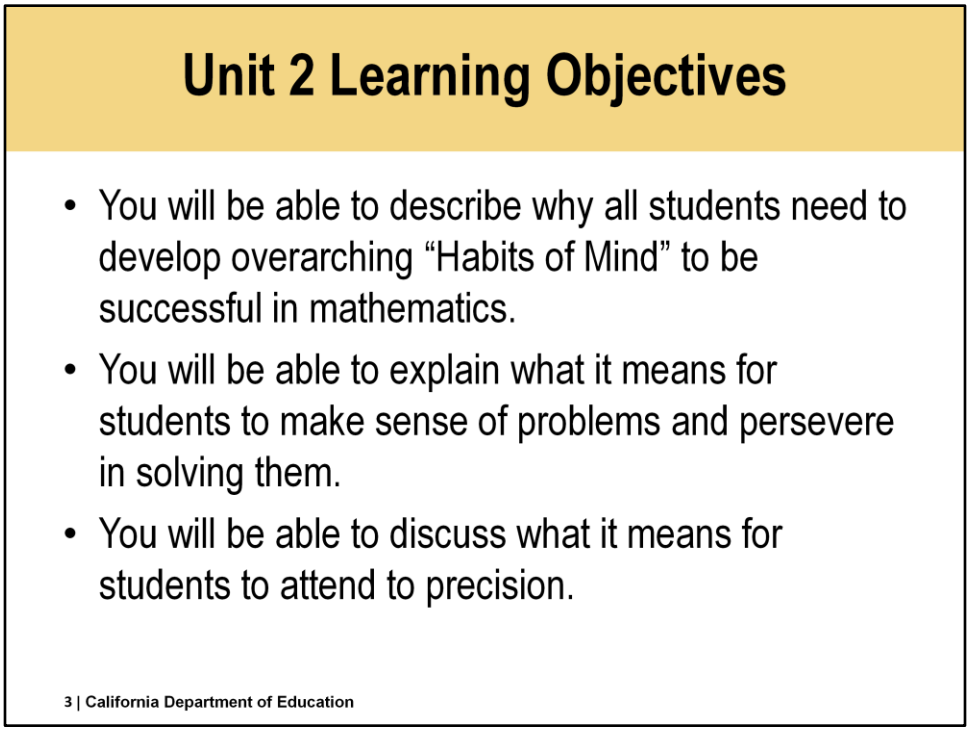
Welcome to Unit 2.



Talking Points:

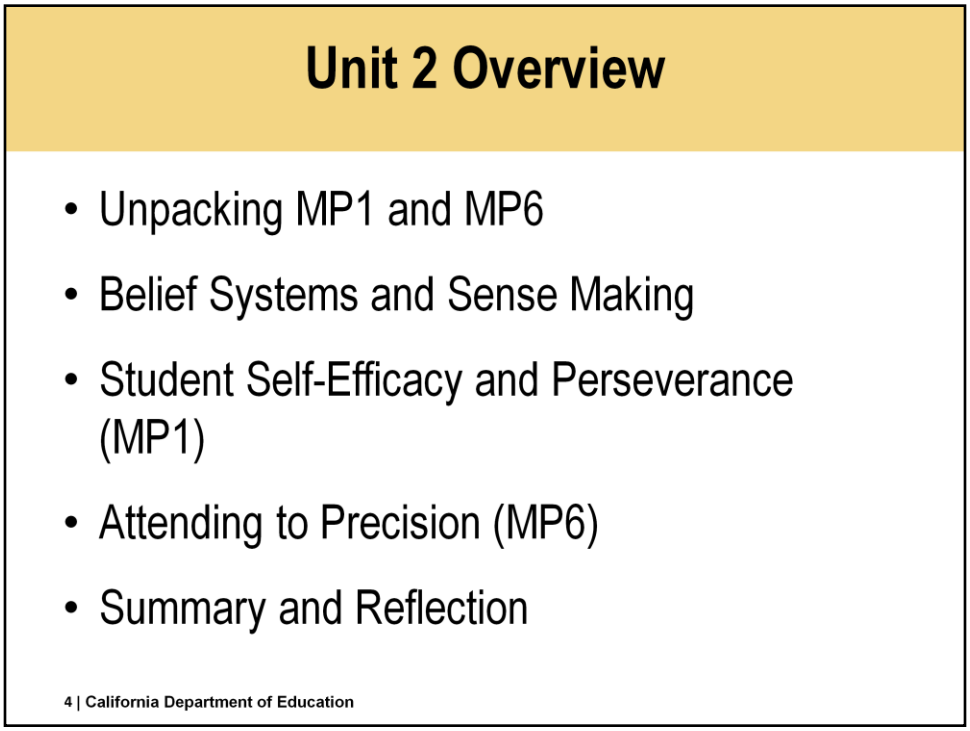
- In this unit, we will review the "Overarching Habits of Mind" practices; MP1 and MP6.

Facilitator Note: Refer participants to the "CCSS Mathematical Practices" handout to keep on hand as a visual reference throughout the training (**Handout 2.0**).



Talking Points:

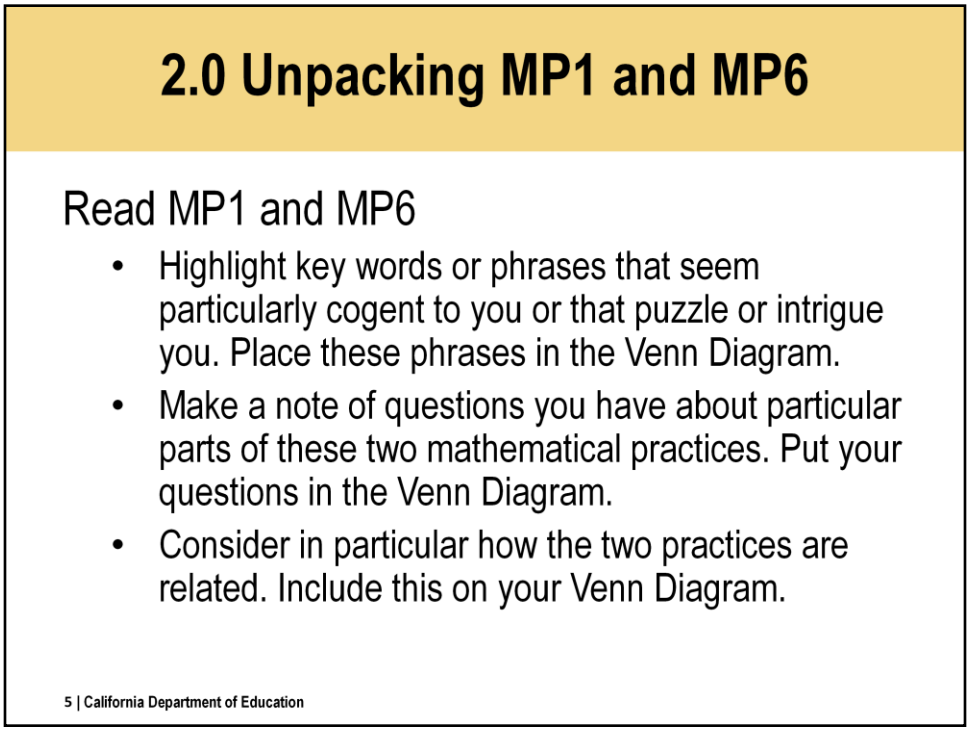
- *What habits exemplify a mathematically proficient student?*
- This unit examines MP1 and MP6; the two standards that relate to "Habits of Mind" — making sense of problems, persevering to a solution, and attending to precision. These principles define what all students must do to become proficient users of mathematics. MP1 and MP6 serve as an umbrella for the other practice standards. In order for the other SMP to be meaningful, the overarching Habits of Mind must be in force.
- You will deepen your understanding of the various aspects of each standard as you work through this unit.



Talking Points:

- These are the sections in Unit 2.

[Review slide content]

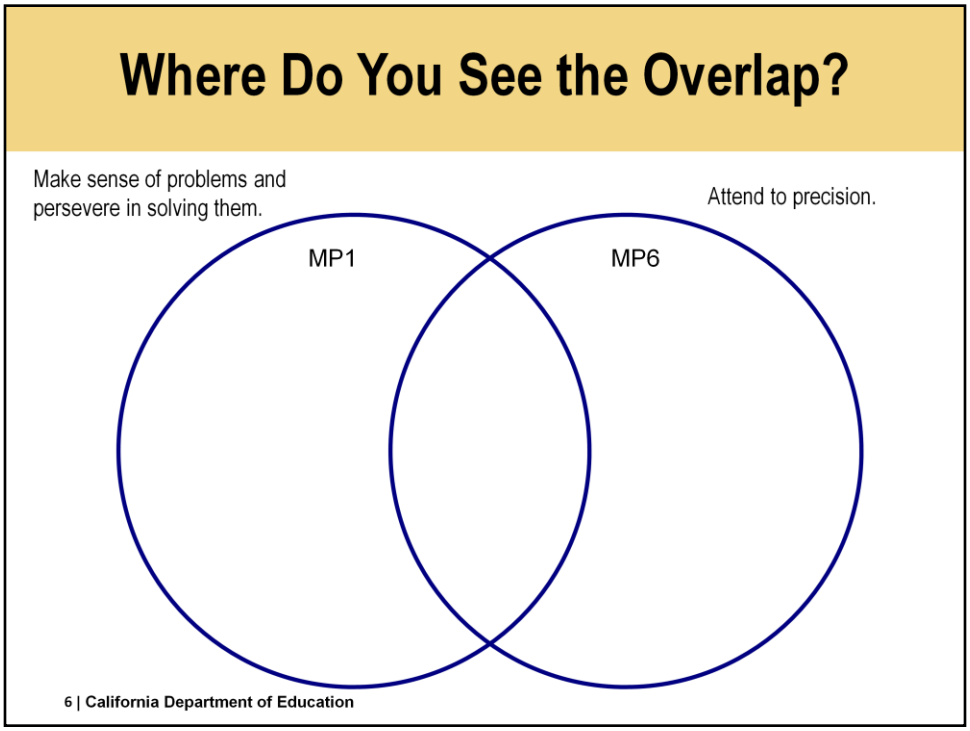


Talking Points:

- MP1 and MP6 are important in every aspect of teaching and learning mathematics. The purpose of these standards is to build within students the sense that they can successfully "do" mathematics and build precision in their use of mathematical symbols, units, and language. These two standards are the Habits of Mind that students need to use in solving any mathematics problem.

Facilitator Notes:

- Refer participants to the Venn Diagram **(Handout 2.0)**, also shown on next slide, for participants to use as they read MP1 and MP6 **(Handout 2.0.1)**.



Facilitator Notes:

- Have participants complete the diagram and identify overlapping areas.

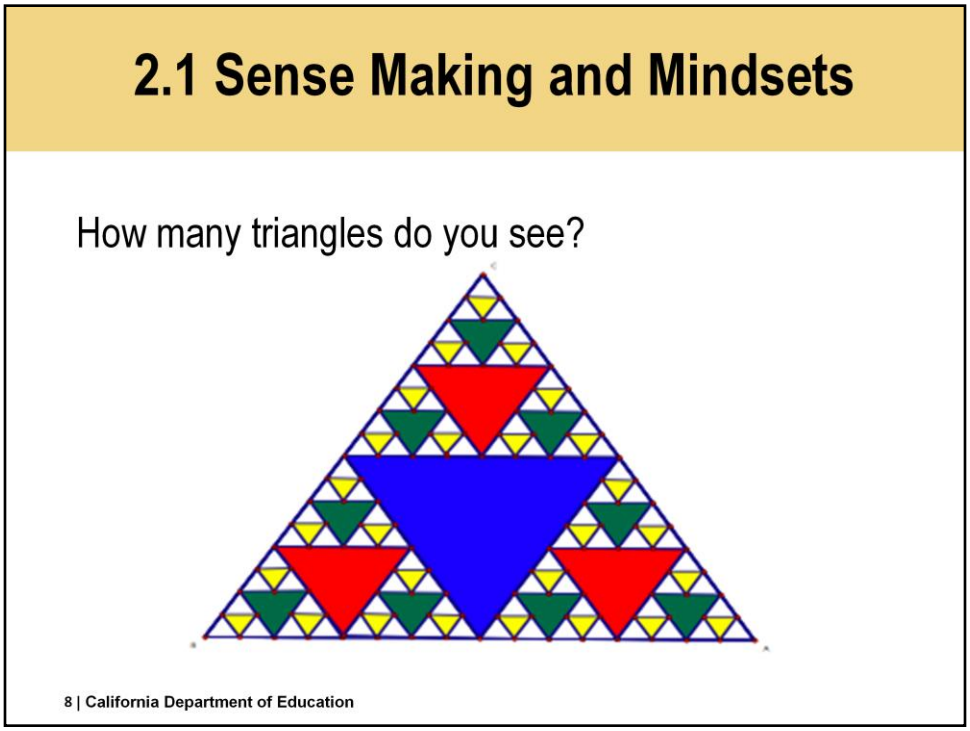
[discussion questions on next slide]

Small Group Discussion

- Compare your Venn diagram with others in your group and discuss the following:
  - What key words or phrases did you highlight and why are they important to you?
  - What questions do you have?
  - How are these standards related to each other?
  - What strategies do you use to help students make sense of problems and persevere in solving them?

Facilitator Notes: If the following points are not addressed in the discussion, be sure to address them:

- MP1 and MP6 are important in every aspect of teaching and learning mathematics.
- Their purpose is to build within students the sense that they can successfully "do" mathematics and build precision in their use of mathematical symbols, units, and language.
- These two standards are the Habits of Mind that students need to use in solving any mathematics problem, and as such, should be visible in every mathematics lesson.



Facilitator Notes:

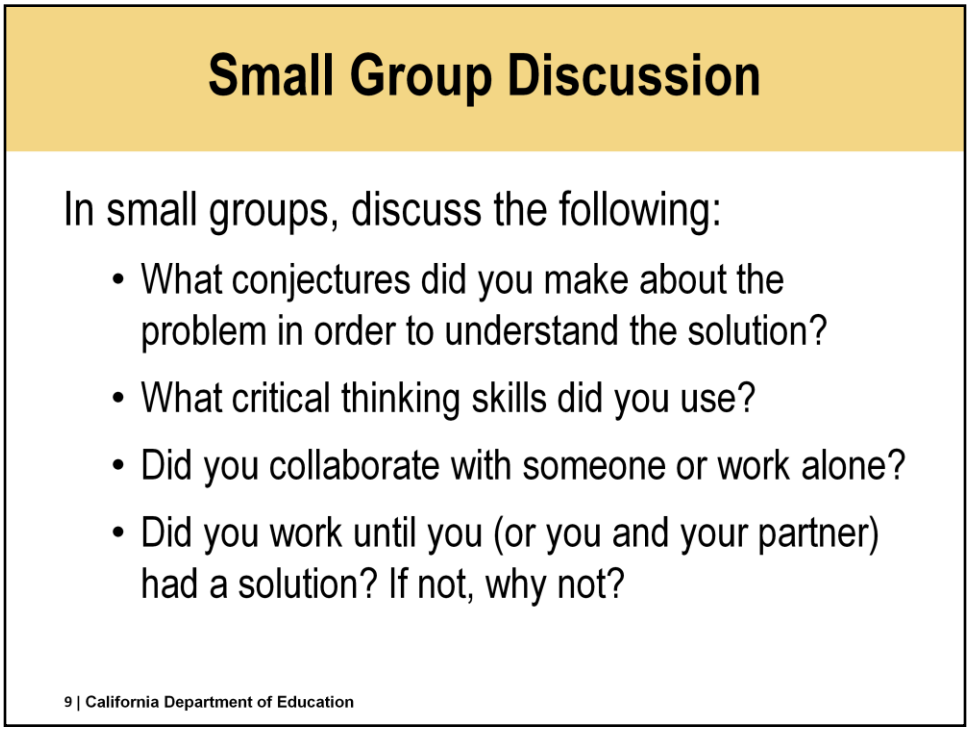
Refer participants to their paper copy of the problem (**Handout 2.1.1**). Ask them to work on this problem alone or with a partner or small group.

Talking Points:

- The diagram shows a Sierpinski's Triangle, and is an example of a fractal.
- A fractal is a geometric pattern that is repeated in ever-smaller scale to produce irregular shapes and surfaces that cannot be represented by classical geometry.

Facilitator Notes:

- Have participants keep track of their thinking by recording how they make sense of the problem and what makes them persevere in solving it.
- Share out solutions using a document camera if possible, choosing solutions that show a variety of strategies; from simplest to more complex, including some that have misconceptions (if there are any) that you want to address.



Facilitator Notes:

Following the small group discussion, ask for volunteers to give highlights of each point. In particular, dwell on the last point.

Talking Points:

- We want to probe the notion of "productive disposition" and the belief in one's own ability and tie that notion to persevering to a solution.

Teacher Mindsets

In order for students to have the desire to make sense of a problem and persevere in finding a solution, students need to believe that the problem is of value, of interest, and that he/she can be successful in doing it. The teacher should also believe the student can successfully do the problem.

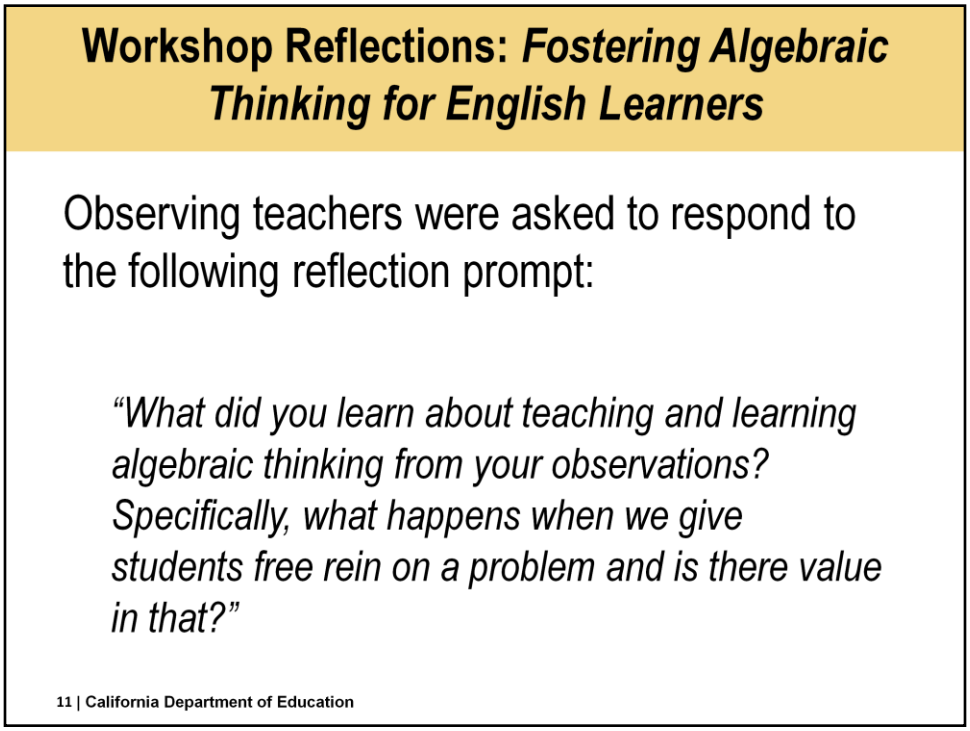
- ***How do you make visible your belief in your students' abilities to complete a task?***

Talking Points:

- To get to a point where a student wants to make sense of a problem and will persevere in finding a solution, the student should believe that it is of value, it is of interest, and that he or she can be successful in doing it. The teacher should also believe the student can successfully do mathematics.

Facilitator notes:

- **Discuss** the bulleted question.



Facilitator Notes:

- Review slide to set the stage for what follows.
- Refer participants to the "Teacher Reflections" document (**Handout 2.1.2)** before advancing to the next slide.

Talking Points:

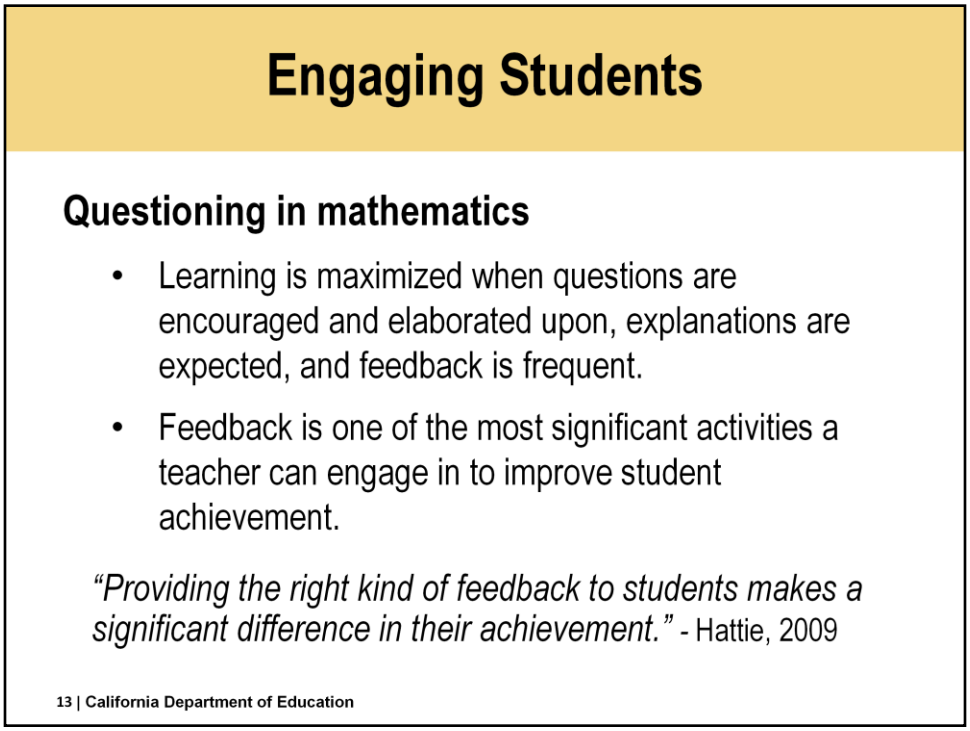
- Refer to the "Teacher Reflections" handout for quotes from teachers who engaged in a two-year professional learning workshop called *Fostering Algebraic Thinking for English Learners,* which was based on the publication, *Fostering Algebraic Thinking*.
- In the last session of the workshop, teachers visited their peers' classrooms and observed students working collaboratively on a common problem.
- In every classroom, the students successfully solved the problem. The visiting teachers sat with groups of students and listened to the students communicate their thinking to each other. The rule was that no one could tell the students what to do; they could only ask probing questions to help the students deepen their mathematical thinking.

Sense Making - Small Group Discussion

- How are the teachers' responses connected to students' sense making?
- What learning did these teachers gain from the collaborative workshop experience?
- How is this learning made visible in their writing?

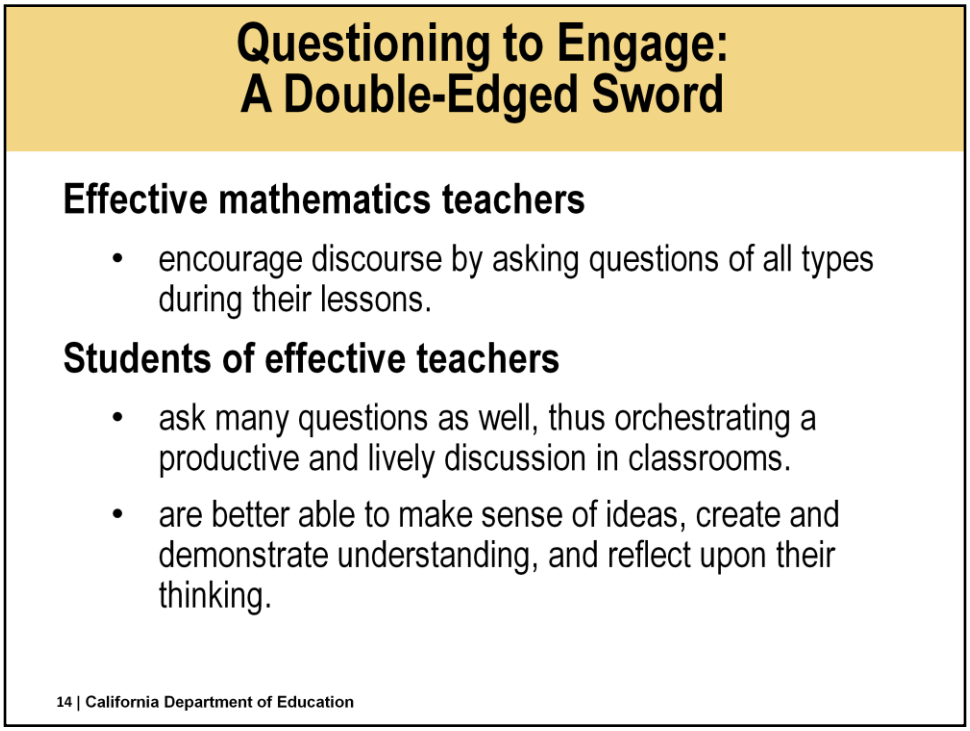
Facilitator Notes:

- Have participants read the reflections. In small groups, respond to the question on the slide.
- Listen to the small group discussions and cull out aspects to share with the whole group.



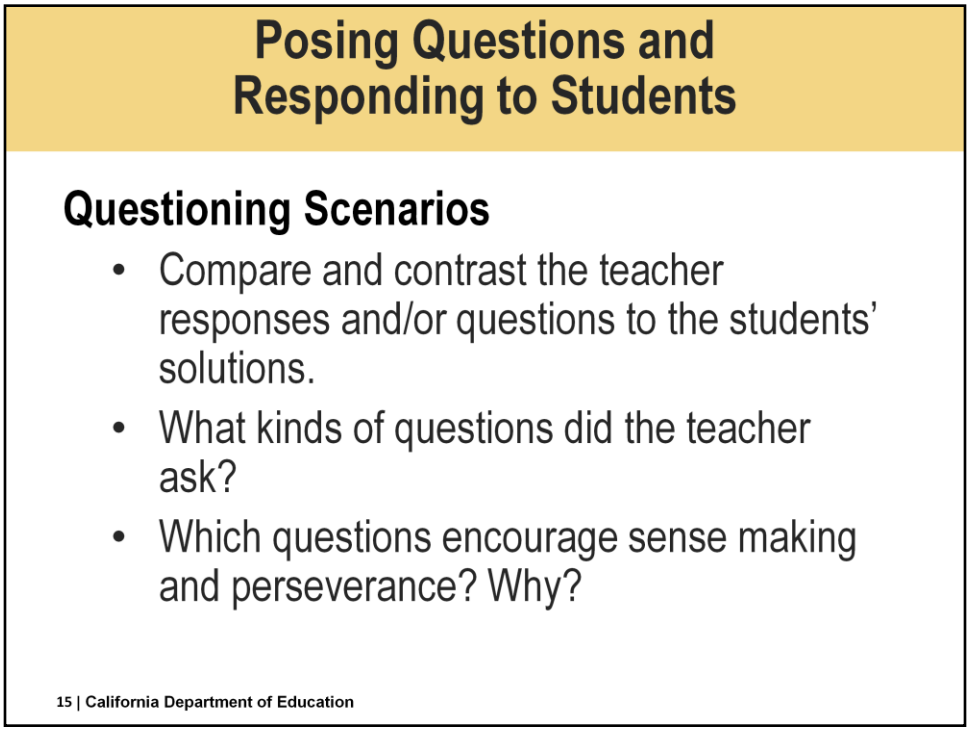
Talking Points:

- The CCSS emphasize the importance of developing mathematical language and communication in order to understand concepts rather than merely following a sequence of procedures. Effective mathematics teachers encourage this kind of discourse by asking questions of all types during their lessons.
- Students of effective teachers ask many questions as well, which orchestrates a more productive and lively discussion in classrooms. Students who engage in discussion are better able to make sense of ideas, create and demonstrate understanding, and reflect upon their thinking.



Talking Points:

- Questioning is a double-edged sword because it is expected of both teachers and students.
- To have a student-centered classroom requires students to ask and answer questions among themselves, not just via the teacher.



Facilitator Notes:

Refer participants to "Posing Questions and Responding to Students" (**Handouts 2.1.3–2.1.6**).

Talking Points:

- Refer to the grade span scenarios in handouts 2.1.3 through 2.1.6. Use the scenario that corresponds to your grade level.
- In each scenario a question is asked, the student answers, followed by responses to the student by different teachers.
- Compare and contrast the teacher responses and questions.
- Focus on the types of questions asked by the teacher that push the student to make sense of the problem and persevere in finding a solution.

Questions that Engage

Listen to the type of questions the teacher asks to push the students' thinking in the video titled, "The Teacher's Role".

Video available at http://vimeo.com/10774338

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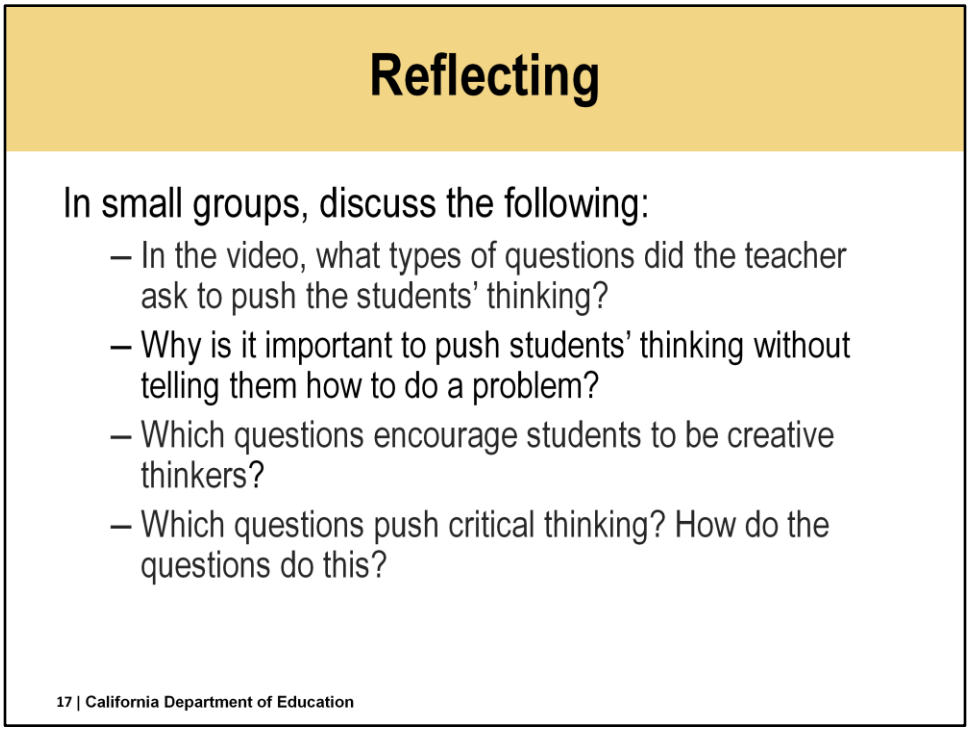
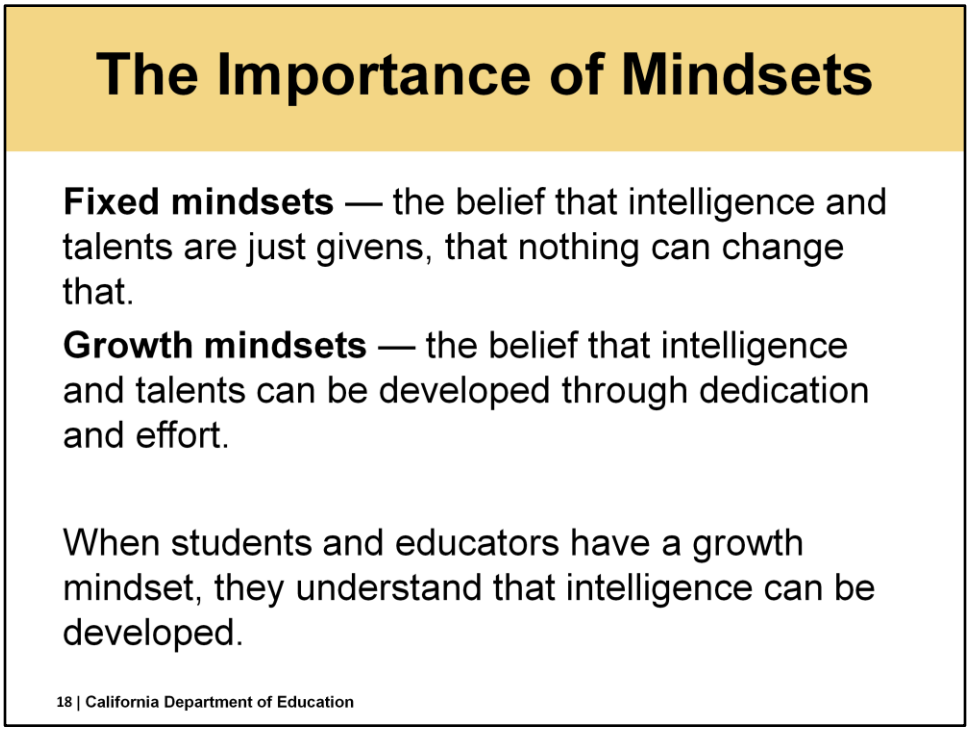
Facilitator Notes:

- Link to the video at the address shown on the slide.
- Ask participants to pay attention to the teacher's questioning strategies.



Facilitator Notes:

- After the small group reflection, ask each group to share some key ideas to the whole group.
- Scenarios should not be skipped.



Talking Points:

- Although research on learning tends to focus on instructional strategies related to subject matter, students' mindsets have a significant effect on their success or failure in school.
- On one end of the spectrum are **fixed mindsets** — the belief that intelligence and talents are just givens, that nothing can change that. Students may believe that no matter how hard they try, they will fail. On the other end of the spectrum are **growth mindsets** — the belief that intelligence and talents can be developed through dedication and effort.
- When students and educators have a growth mindset, they understand that intelligence can be developed. Students focus on improvement instead of worrying about how smart they are.
- They work hard to learn more and get smarter. Based on research by Stanford University's Carol Dweck, Lisa Blackwell, and their colleagues, students who learn the growth mindset show greater motivation in school.

2.2 Student Self-Efficacy and Perseverance (MP1)

Research makes clear the connection between effort and achievement —believing you can often makes it so. This belief can give students the resiliency to "pick themselves up, dust themselves off, and try all over again."

Share with a partner an experience you had as a student in which a teacher convinced you that you could do something that you believed was difficult for you.

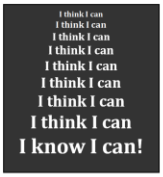
Talking Points:

- Although research on learning tends to focus on instructional strategies related to subject matter, students' beliefs and attitudes have a significant effect on their success or failure in school. Students growing up amid challenges can develop an attitude that "failure is just around the corner, no matter how hard they work."
- The Sierpinski's Triangle task that you completed earlier is thought to be accessible at many levels and appealing to many learners. However, will that always be the case?
- To get to a point where a student wants to make sense of a problem and will persevere in finding a solution, the student should believe that it is of value, it is of interest, and that he or she can be successful in doing it. The teacher should also believe the student can successfully do mathematics.

Student Self-Efficacy and Perseverance (cont.)

Self-Efficacy: Belief in one's ability to achieve a goal or a task.

- **Effort** usually refers to whether a student tries hard, asks for help, and/or participates in class Involves choice, is rooted in beliefs, and is influenced by feedback.
- **Interest** is a cognitive and affective relationship between a student and a particular subject that varies depending on the type of interest being described.

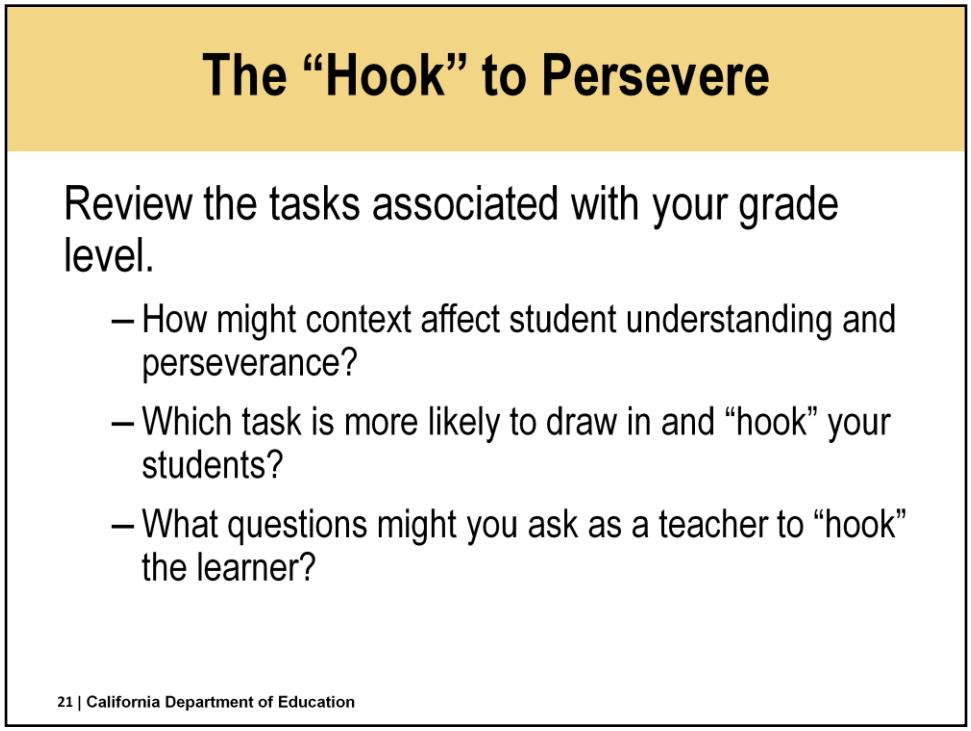


Talking Points:

- A students' self-efficacy is the belief in his or her ability to achieve a goal or a task. Students with a strong self-efficacy are likely to persist when faced with a challenging task, whereas a student with low self-efficacy is more likely to give up. The teachers' belief in the efficacy of their students is the first step toward student success.
- The effort that a student makes to accomplish a task is strongly influenced by interest and context. Interest can motivate students with low self-efficacy to see tasks through.
- **Effort** usually refers to whether a student tries hard, asks for help, and/or participates in class Involves choice, is rooted in beliefs, and is influenced by feedback.
- **Interest** is a cognitive and affective relationship between a student and a particular subject that varies depending on the type of interest being described. Interest can hold a student's attention, encourage effort, and support learning. It can also enhance strategic processing. ALL students can be interested in a wide variety of things.

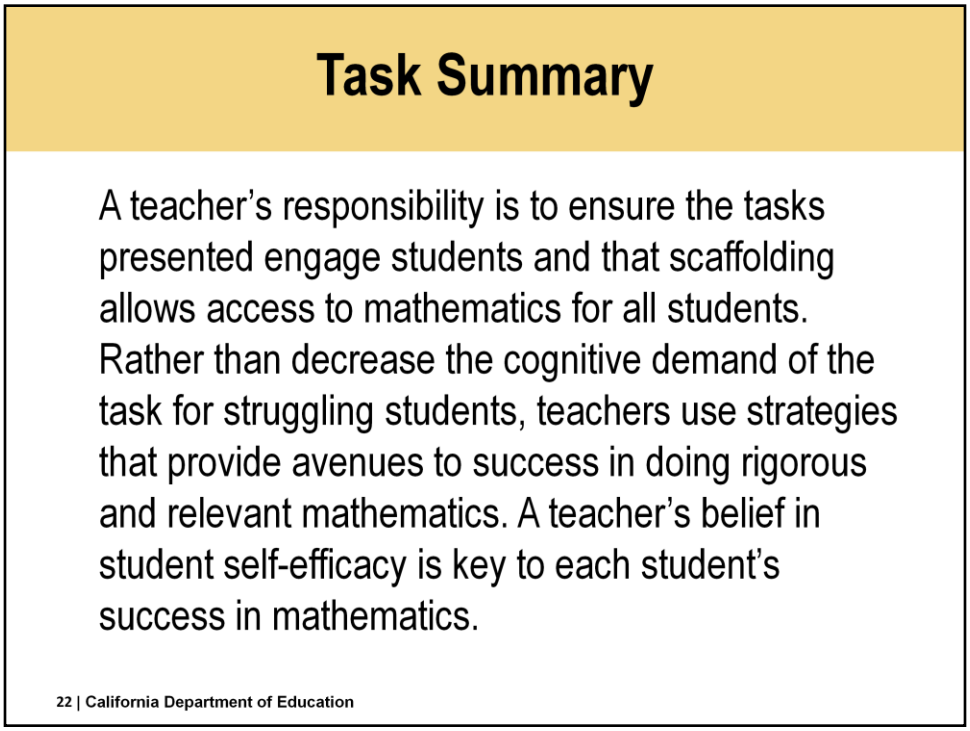
Facilitator notes:

- Ask participants to reflect on how they inspire effort and use interest in their classrooms.



Talking Points:

- There are four handouts for this section in your participant packet; one for each grade level (**Handouts 2.2.1 through 2.2.4**).
- Select the appropriate grade level for your group, or divide into grade level groups (Primary, Upper elementary, Middle school, High school)
- Instructions:
  - You have been assigned a grade level series of tasks. In each set, the same mathematical concept is addressed.
  - As you complete the activity, compare the tasks within your grade span concept and consider how context might affect student understanding and perseverance.
  - Consider which task is more likely to draw in and "hook" your students, encouraging them to persevere to find a solution. Consider also the questions you might ask as a teacher to "hook" the learner.



Facilitator Notes:

- Read the summary on the slide (or have a participant read the summary).
- Discuss the importance of relevance in engaging students in doing mathematics. Why are context-free tasks less likely to engage?
  - Teachers will respond that it takes more time to do contextual tasks. Engage them in a discussion of "covering the material" vs. "learning the mathematics."
- The discussion should also include the idea of scaffolding as a method to get ALL students to a high cognitive level, not as a way to water down the mathematics for access. Access includes the scaffolding to reach the high cognitive demand expected of all students.

Optional Activity

Work in small groups with other teachers who teach the same course or grade level on the following:

- Take a standard's based grade level task that you plan to use in the near future.
- Alter it to include a "hook" that will engage all of your students in doing mathematics.
- Include in the task strategies that will make student thinking visible.

Facilitator Notes: This is an optional activity and requires that the participants have grade-level tasks on hand.

- Divide the participants into grade level or same course groups.
- Have them take a task and revise it to include a "hook" that will engage students in doing mathematics.
- Give participants time to use what they have learned. This is not a fully developed lesson plan, but one task that would occur within the lesson.

Reflection and Discussion

Discuss in small groups:

- What role might student communication and collaboration play in "hooking" the students in your task?
- How would you modify your grade level task to meet the needs of your student population?
  - In what ways would you scaffold your task to support your English learners to meet language demands?
  - In what ways will you scaffold your task to support your students with disabilities gain access to the task?
  - How will you differentiate for gifted students?

Facilitator Notes:

- Review slide instructions.
- Upon conclusion of the activity, have participants share out with the whole group.

2.3 Attend to Precision

One example of attending to precision is in analyzing the **necessity** and **sufficiency** of mathematical definitions.

- **Necessity*:*** A necessary condition of a statement must be satisfied for the statement to be true.
- **Sufficiency:** A sufficient condition is one that, if satisfied, assures the statement's truth.

Facilitator Notes:

- Explain precision by defining the terms on the slide.

Talking Points:

- Precision in mathematics is often thought of in terms of accuracy of measurements. This is an important part of precision in mathematics, but “attend to precision” also refers to the way in which we use the language and symbols of mathematics, in particular the clarity of language and use of definitions.

An Example of Precision

Statement about a cat: "*A cat is a mammal*."

- This statement satisfies the **necessity** condition because the condition of being a mammal is necessary for cats.
- The statement does not satisfy the **sufficiency** condition because there are mammals that are not cats.

What are some other examples?

Talking Points:

- Although the cat example appears to be fairly simple, in mathematics students often assume necessary conditions are also sufficient, especially in the ways they make their statements.
- Let's investigate this phenomena by looking at some actual student work!

Focusing on Mathematical Statements

Refer to the "Focusing on Mathematical Statements" handout.

- Read statements about rectangles written by grade-level teachers in an online geometry course as they learned about statements.
- What do you notice about their statements?

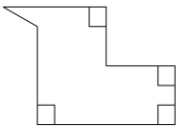
Facilitator Notes:

- Refer participants to the "Focusing on Mathematical Statements" handout (**Handout 2.3.1)**
- Give participants time to read the statements and think about the questions at the bottom of the page:
  - ✓ Are the statements precise?
  - ✓ Are the conditions stated necessary?
  - ✓ Are the conditions stated sufficient?

Analyses of Teachers' Statements: Special Education and 6th Grade Teacher

"If a figure has four right angles, then it is a rectangle."

Analysis: The counterexample is given below. This is a figure with four right angles but is not a rectangle. However, if we look at the converse of the statement, "if a figure is a rectangle, then the figure has four right angles", we find that this is a correct conditional statement.



Facilitator Notes:

- Review slide.
- Ask teachers to construct another figure with 4 right angles that is not a rectangle.

Analyses of Teachers' Statements: 2nd and 4th Grade Teacher

"If a figure is a rectangle, then it is is a four-sided polygon with four right angles."

Analysis: The converse of the statement, "if a four-sided polygon has four right angles, then the figure is a rectangle," is also a true statement. There is no question that this describes a rectangle. However, is this precise information?

- Is it possible to draw a four-sided polygon with three right angles that is not a rectangle?
- What about a four-sided polygon with two right angles that is not a rectangle?

Facilitator Notes:

- Have participants reflect on this analysis.
- Give time for small group discussion.
- Move to next slide for continued discussion of this analysis.

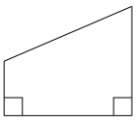
How Many Right Angles Do We Need?

Is it possible to draw a four-sided polygon with three right angles that is not a rectangle?

- In the diagram below there are three right angles and three sides. To close the polygon, we will automatically make a rectangle.

What about a four-sided polygon with two right angles that is not a rectangle?

- Counterexample: The figure below is a four-sided polygon with two right angles, but it is not a rectangle.



It would be precise to say, "a rectangle is a four-sided polygon with three right angles."

Facilitator Notes:

- Review slide and have participants reflect on this analysis.

Analyses of Teachers' Statements: 7th Grade Teacher

"If a shape is a rectangle, then it is a parallelogram with at least two right angles."

Analysis:

- **Case 1:** If I make the two right angles the base angles, I know that the opposite angles are congruent since it is a parallelogram, so all of the angles are right angles.

- **Case 2:** If I make the two right angles the diagonal angles, I know that two adjacent angles are supplementary since it is a parallelogram, so all of the angles are right angles. So this is a rectangle.

Facilitator Notes:

- Have participants reflect on this analysis.

Beyond Sufficiency

- The 7th grade statement has the necessary conditions for being a rectangle, but goes beyond what is sufficient. Why? Let's change the statement to:

  "If a shape is a rectangle, then it is a parallelogram with one right angle."

- If angle A is the right angle, then we know angle C is also a right angle because opposite angles of a parallelogram are congruent. Then using the thinking we used in the last example, consecutive angles of a parallelogram are supplementary, so angles B and D are also right angles.

Conclusion: The statement **"a rectangle is a parallelogram with one right angle"** is a precise statement.

Facilitator Notes:

- Review slide and have participants reflect on this analysis.

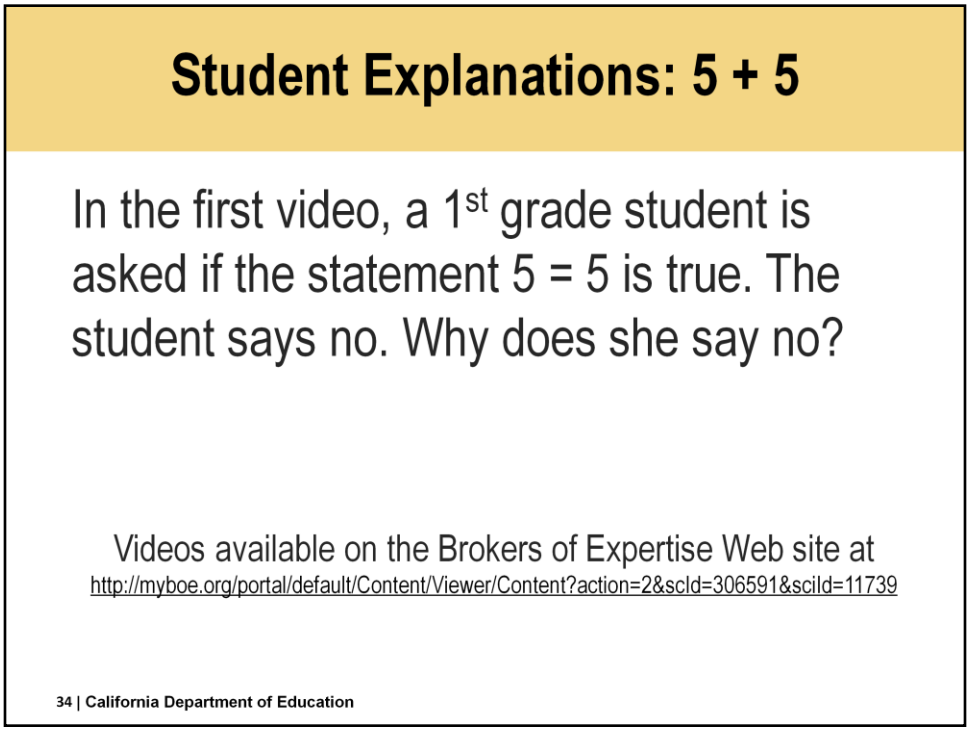
An Example of Students Attending to Precision

Following are three video clips of students explaining their understanding of the "equals" sign.

Listen carefully to their thinking and reflect on the question that follows each clip.

Talking Points:

- On the next three slides are videos in which students explain their understanding of the "equals" sign.
- Listen carefully to the students' statements.



Facilitator Notes:

- Before starting the video clip, have the participants read the question at the bottom.
- After watching, discuss the question as a group.
- Videos available on the Brokers of Expertise Web site at http://myboe.org/portal/default/Content/Viewer/Content?action=2&scId=306591&scId=11739

Student Explanations: 3 + 5 = 5 + 3

In the second video, a 3rd grade student is asked if the statement 3 + 5 = 3 + 5 is true. The student says it is false. What is his thinking?

Facilitator Notes:

- Before starting the video clip, have the participants read the question.
- After watching, discuss the question as a group.

Student Explanations: 3 + 5 = 8 and 8 = 3 + 5

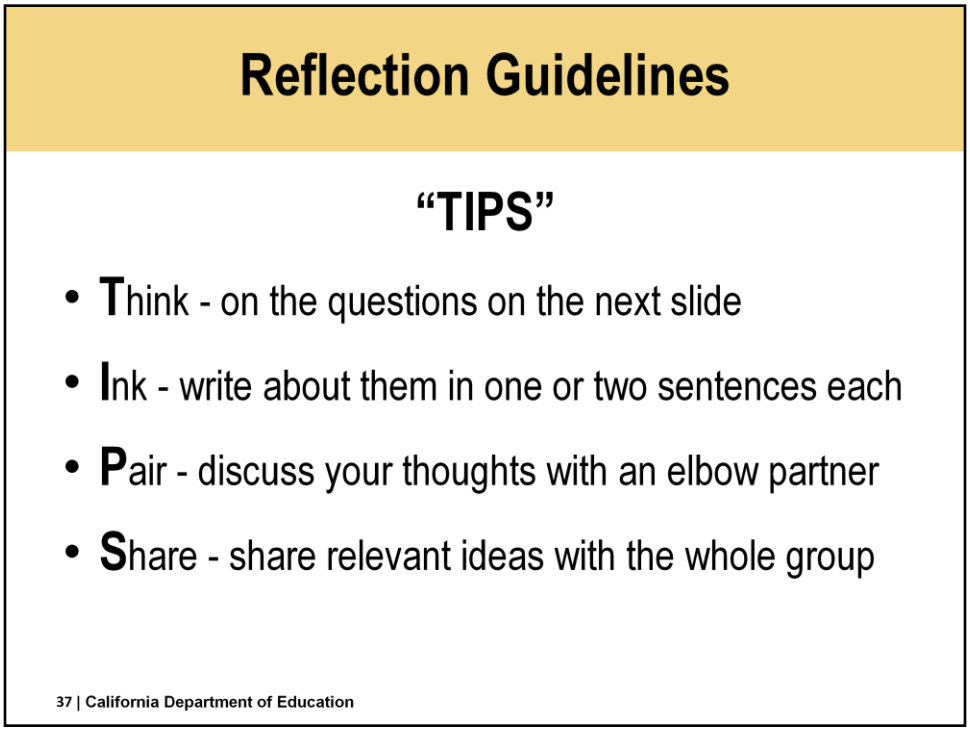
In the third video, a 2nd grade student is asked if 3 + 5 = 8 is the same as 8 = 3 + 5. She says that it depends on whether it makes sense to kids or if it is what the teacher wants. How might students adopt this idea?

Facilitator Notes:

- Before starting the video clip, have the participants read the question.
- After watching, discuss the question as a group.

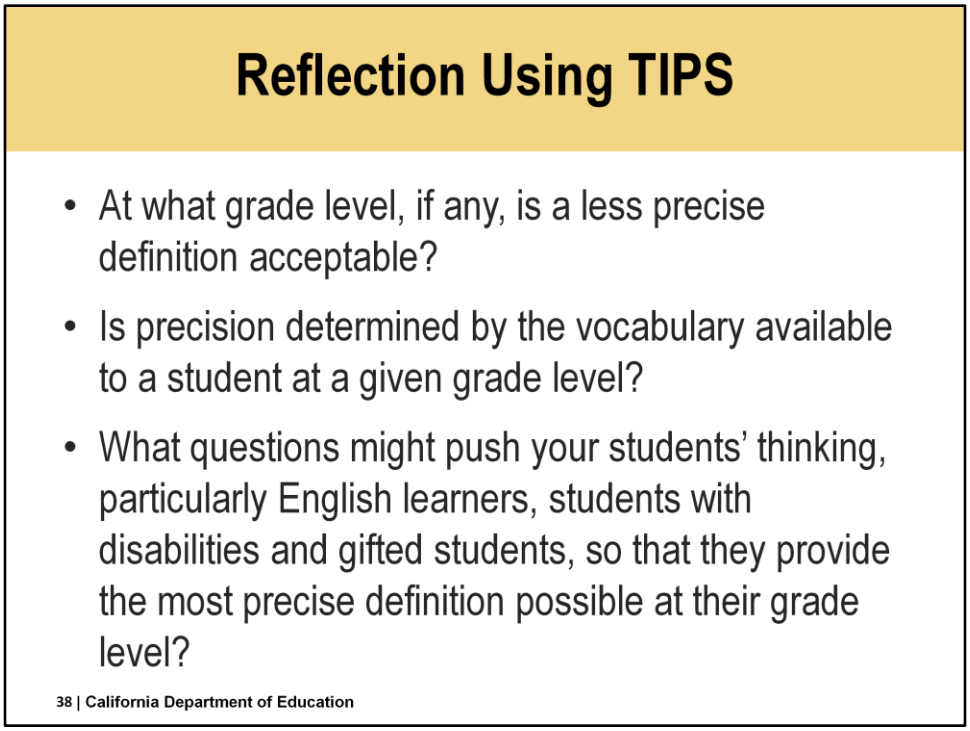
Talking Points:

- The third student bases her answer on whether or not it has to make sense or be what the teacher wants to hear. We can be certain that no teacher intentionally sets up students for misunderstanding. A teacher's "habit of mind" should be tuned to listen to student thinking; this ensures what we think we are teaching is what they are learning.



Facilitator Notes:

- Review reflection guidelines with participants.



Facilitator Notes:

- Display slide during reflection activity.

2.4 Summary and Reflection

- **MP1:** Making sense and persevering are habits of mind needed by all students to be successful learners of mathematics. Before students can engage in mathematics, they need to make sense of what they are being asked to consider.

- **MP6:** Precision refers to the accuracy with which students use mathematical language and symbols as well as precision in measurement.

All students must have access to the language and symbols of mathematics. They can learn to talk in the language of mathematics when given the opportunity; all students have this potential.

Facilitator Notes:

- Review slide with participants. End with the statement below:

Talking Points:

- It is the responsibility of teachers to use available tools to give ALL students access.

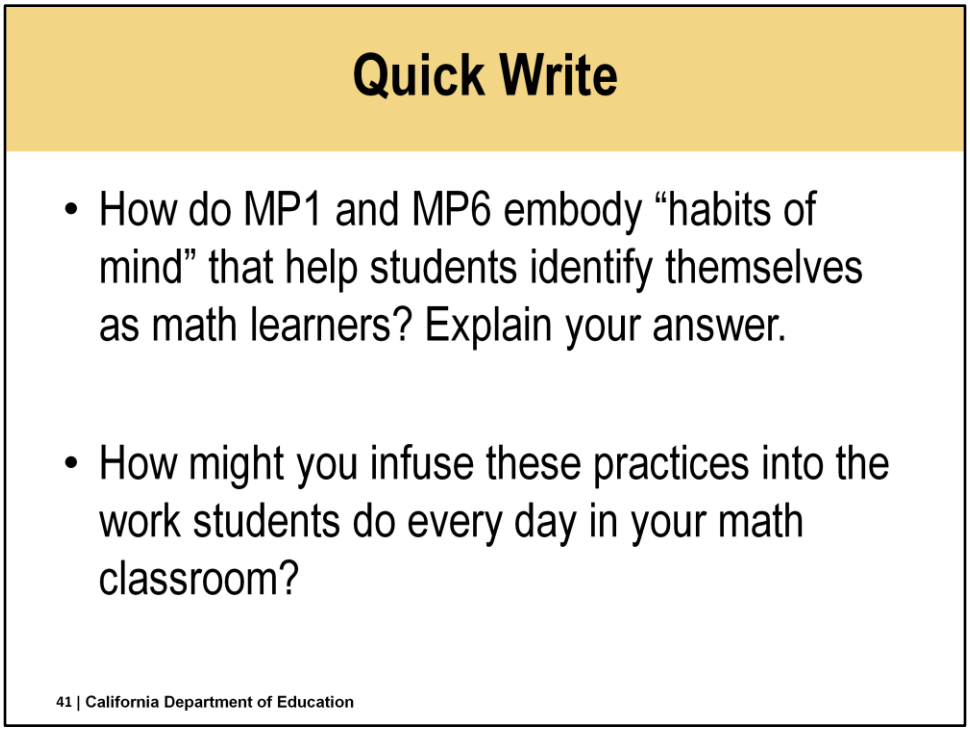
Revisiting Objectives

At this point, you should be able to:

- Describe why, to be successful in mathematics, all students need to develop overarching habits of mind.
- Explain what it means for students to make sense of problems and persevere in solving them.
- Discuss what it means for students to attend to precision.

Facilitator Notes:

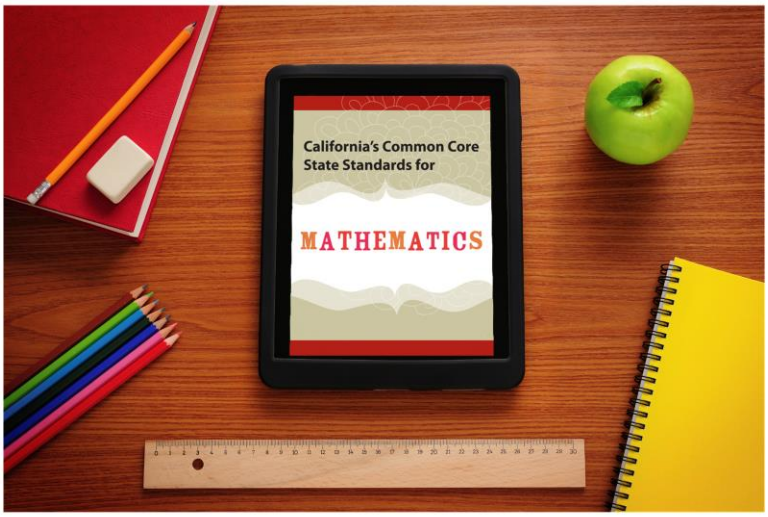
- Review slide.



Notes to Facilitator:

- Refer participants to journal and have them write responses to reflection questions on slide.
- Engage in discussion as time allows.

California's Common Core State Standards for Mathematics



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(TRANSITION SLIDE)