Kindergarten through Grade Twelve
Standards for Mathematical Practice

Unit 2

Overarching Habits of Mind: MP1 and MP6

CALIFORNIA DEPARTMENT OF EDUCATION
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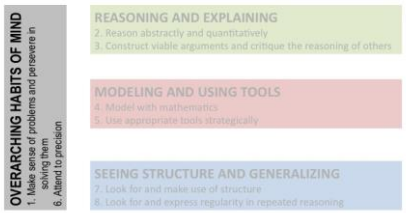
CCSS Mathematical Practices



Unit 2 Learning Objectives

- You will be able to describe why all students need to develop overarching "Habits of Mind" to be successful in mathematics.
- You will be able to explain what it means for students to make sense of problems and persevere in solving them.
- You will be able to discuss what it means for students to attend to precision.

Unit 2 Overview

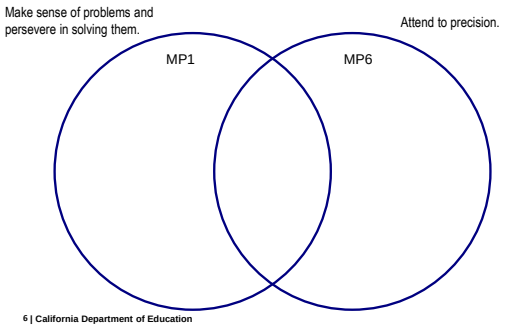
- Unpacking MP1 and MP6
- Belief Systems and Sense Making
- Student Self-Efficacy and Perseverance (MP1)
- Attending to Precision (MP6)
- Summary and Reflection

2.0 Unpacking MP1 and MP6

Read MP1 and MP6

- Highlight key words or phrases that seem particularly cogent to you or that puzzle or intrigue you. Place these phrases in the Venn Diagram.
- Make a note of questions you have about particular parts of these two mathematical practices. Put your questions in the Venn Diagram.
- Consider in particular how the two practices are related. Include this on your Venn Diagram.

Where Do You See the Overlap?

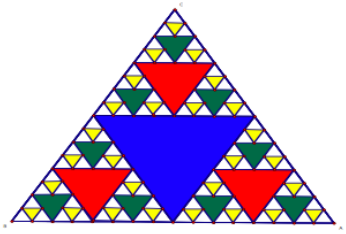


Small Group Discussion

- Compare your Venn diagram with others in your group and discuss the following:
  - What key words or phrases did you highlight and why are they important to you?
  - What questions do you have?
  - How are these standards related to each other?
  - What strategies do you use to help students make sense of problems and persevere in solving them?

2.1 Sense Making and Mindsets

How many triangles do you see?



Small Group Discussion

In small groups, discuss the following:

- What conjectures did you make about the problem in order to understand the solution?
- What critical thinking skills did you use?
- Did you collaborate with someone or work alone?
- Did you work until you (or you and your partner) had a solution? If not, why not?

Teacher Mindsets

In order for students to have the desire to make sense of a problem and persevere in finding a solution, students need to believe that the problem is of value, of interest, and that he/she can be successful in doing it. The teacher should also believe the student can successfully do the problem.

- ***How do you make visible your belief in your students' abilities to complete a task?***

Workshop Reflections: *Fostering Algebraic Thinking for English Learners*

Observing teachers were asked to respond to the following reflection prompt:

"What did you learn about teaching and learning algebraic thinking from your observations? Specifically, what happens when we give students free rein on a problem and is there value in that?"

Sense Making - Small Group Discussion

- How are the teachers' responses connected to students' sense making?
- What learning did these teachers gain from the collaborative workshop experience?
- How is this learning made visible in their writing?

Engaging Students

Questioning in mathematics

- Learning is maximized when questions are encouraged and elaborated upon, explanations are expected, and feedback is frequent.
- Feedback is one of the most significant activities a teacher can engage in to improve student achievement.

"Providing the right kind of feedback to students makes a significant difference in their achievement." - Hattie, 2009

Questioning to Engage: A Double-Edged Sword

Effective mathematics teachers

- encourage discourse by asking questions of all types during their lessons.

Students of effective teachers

- ask many questions as well, thus orchestrating a productive and lively discussion in classrooms.
- are better able to make sense of ideas, create and demonstrate understanding, and reflect upon their thinking.

Posing Questions and Responding to Students

Questioning Scenarios

- Compare and contrast the teacher responses and/or questions to the students' solutions.
- What kinds of questions did the teacher ask?
- Which questions encourage sense making and perseverance? Why?

Questions that Engage

Listen to the type of questions the teacher asks to push the students' thinking in the video titled, "The Teacher's Role".

Video available at http://vimeo.com/10774338

Reflecting

In small groups, discuss the following:

- In the video, what types of questions did the teacher ask to push the students' thinking?
- Why is it important to push students' thinking without telling them how to do a problem?
- Which questions encourage students to be creative thinkers?
- Which questions push critical thinking? How do the questions do this?

The Importance of Mindsets

Fixed mindsets — the belief that intelligence and talents are just givens, that nothing can change that.

Growth mindsets — the belief that intelligence and talents can be developed through dedication and effort.

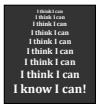
When students and educators have a growth mindset, they understand that intelligence can be developed.

2.2 Student Self-Efficacy and Perseverance (MP1)

Research makes clear the connection between effort and achievement —believing you can often makes it so. This belief can give students the resiliency to "pick themselves up, dust themselves off, and try all over again."

Share with a partner an experience you had as a student in which a teacher convinced you that you could do something that you believed was difficult for you.

Student Self-Efficacy and Perseverance (cont.)



Self-Efficacy: Belief in one's ability to achieve a goal or a task.

- **Effort** usually refers to whether a student tries hard, asks for help, and/or participates in class Involves choice, is rooted in beliefs, and is influenced by feedback.
- **Interest** is a cognitive and affective relationship between a student and a particular subject that varies depending on the type of interest being described.

The "Hook" to Persevere

Review the tasks associated with your grade level.

- How might context affect student understanding and perseverance?
- Which task is more likely to draw in and "hook" your students?
- What questions might you ask as a teacher to "hook" the learner?

Task Summary

A teacher's responsibility is to ensure the tasks presented engage students and that scaffolding allows access to mathematics for all students. Rather than decrease the cognitive demand of the task for struggling students, teachers use strategies that provide avenues to success in doing rigorous and relevant mathematics. A teacher's belief in student self-efficacy is key to each student's success in mathematics.

Optional Activity

Work in small groups with other teachers who teach the same course or grade level on the following:

- Take a standard's based grade level task that you plan to use in the near future.
- Alter it to include a "hook" that will engage all of your students in doing mathematics.
- Include in the task strategies that will make student thinking visible.

Reflection and Discussion

Discuss in small groups:

- What role might student communication and collaboration play in "hooking" the students in your task?
- How would you modify your grade level task to meet the needs of your student population?
  - In what ways would you scaffold your task to support your English learners to meet language demands?
  - In what ways will you scaffold your task to support your students with disabilities gain access to the task?
  - How will you differentiate for gifted students?

2.3 Attend to Precision

One example of attending to precision is in analyzing the **necessity** and **sufficiency** of mathematical definitions.

- **Necessity:** A necessary condition of a statement must be satisfied for the statement to be true.
- **Sufficiency:** A sufficient condition is one that, if satisfied, assures the statement's truth.

An Example of Precision

Statement about a cat: "*A cat is a mammal.*"

- This statement satisfies the **necessity** condition because the condition of being a mammal is necessary for cats.
- The statement does not satisfy the **sufficiency** condition because there are mammals that are not cats.

What are some other examples?

Focusing on Mathematical Statements

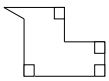
Refer to the "Focusing on Mathematical Statements" handout.

- Read statements about rectangles written by grade-level teachers in an online geometry course as they learned about statements.
- What do you notice about their statements?

Analyses of Teachers' Statements: Special Education and 6th Grade Teacher

"If a figure has four right angles, then it is a rectangle."

Analysis: The counterexample is given below. This is a figure with four right angles but is not a rectangle. However, if we look at the converse of the statement, "if a figure is a rectangle, then the figure has four right angles", we find that this is a correct conditional statement.



Analyses of Teachers' Statements: 2nd and 4th Grade Teacher

"If a figure is a rectangle, then it is is a four-sided polygon with four right angles."

Analysis: The converse of the statement, "if a four-sided polygon has four right angles, then the figure is a rectangle," is also a true statement. There is no question that this describes a rectangle. However, is this precise information?

- Is it possible to draw a four-sided polygon with three right angles that is not a rectangle?
- What about a four-sided polygon with two right angles that is not a rectangle?

How Many Right Angles Do We Need?

Is it possible to draw a four-sided polygon with three right angles that is not a rectangle?

- In the diagram below there are three right angles and three sides. To close the polygon, we will automatically make a rectangle.

What about a four-sided polygon with two right angles that is not a rectangle?

- Counterexample: The figure below is a four-sided polygon with two right angles, but it is not a rectangle.

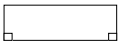
It would be precise to say, "a rectangle is a four-sided polygon with three right angles."

Analyses of Teachers' Statements: 7th Grade Teacher

"If a shape is a rectangle, then it is a parallelogram with at least two right angles."

Analysis:

- **Case 1:** If I make the two right angles the base angles, I know that the opposite angles are congruent since it is a parallelogram, so all of the angles are right angles.



- **Case 2:** If I make the two right angles the diagonal angles, I know that two adjacent angles are supplementary since it is a parallelogram, so all of the angles are right angles. So this is a rectangle.

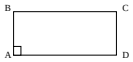
Beyond Sufficiency

- The 7th grade statement has the necessary conditions for being a rectangle, but goes beyond what is sufficient. Why? Let's change the statement to:

  "If a shape is a rectangle, then it is a parallelogram with one right angle."

- If angle A is the right angle, then we know angle C is also a right angle because opposite angles of a parallelogram are congruent. Then using the thinking we used in the last example, consecutive angles of a parallelogram are supplementary, so angles B and D are also right angles.

Conclusion: The statement **"a rectangle is a parallelogram with one right angle"** is a precise statement.



An Example of Students Attending to Precision

Following are three video clips of students explaining their understanding of the "equals" sign.

Listen carefully to their thinking and reflect on the question that follows each clip.

Student Explanations: 5 + 5

In the first video, a 1st grade student is asked if the statement 5 = 5 is true. The student says no. Why does she say no?

Videos available on the Brokers of Expertise Web site at
http://myboe.org/portal/default/Content/Viewer/Content?action=2&scId=306591&sciId=11739

Student Explanations: 3 + 5 = 5 + 3

In the second video, a 3rd grade student is asked if the statement 3 + 5 = 3 + 5 is true. The student says it is false. What is his thinking?

Student Explanations: 3 + 5 = 8 and 8 = 3 + 5

In the third video, a 2nd grade student is asked if 3 + 5 = 8 is the same as 8 = 3 + 5. She says that it depends on whether it makes sense to kids or if it is what the teacher wants. How might students adopt this idea?

Reflection Guidelines

"TIPS"

- **T**hink - on the questions on the next slide
- **I**nk - write about them in one or two sentences each
- **P**air - discuss your thoughts with an elbow partner
- **S**hare - share relevant ideas with the whole group

Reflection Using TIPS

- At what grade level, if any, is a less precise definition acceptable?
- Is precision determined by the vocabulary available to a student at a given grade level?
- What questions might push your students' thinking, particularly English learners, students with disabilities and gifted students, so that they provide the most precise definition possible at their grade level?

2.4 Summary and Reflection

- **MP1:** Making sense and persevering are habits of mind needed by all students to be successful learners of mathematics. Before students can engage in mathematics, they need to make sense of what they are being asked to consider.
- **MP6:** Precision refers to the accuracy with which students use mathematical language and symbols as well as precision in measurement.

All students must have access to the language and symbols of mathematics. They can learn to talk in the language of mathematics when given the opportunity; all students have this potential.

Revisiting Objectives

At this point, you should be able to:

- Describe why, to be successful in mathematics, all students need to develop overarching habits of mind.
- Explain what it means for students to make sense of problems and persevere in solving them.
- Discuss what it means for students to attend to precision.

Quick Write

- How do MP1 and MP6 embody "habits of mind" that help students identify themselves as math learners? Explain your answer.
- How might you infuse these practices into the work students do every day in your math classroom?

California's Common Core State Standards for Mathematics