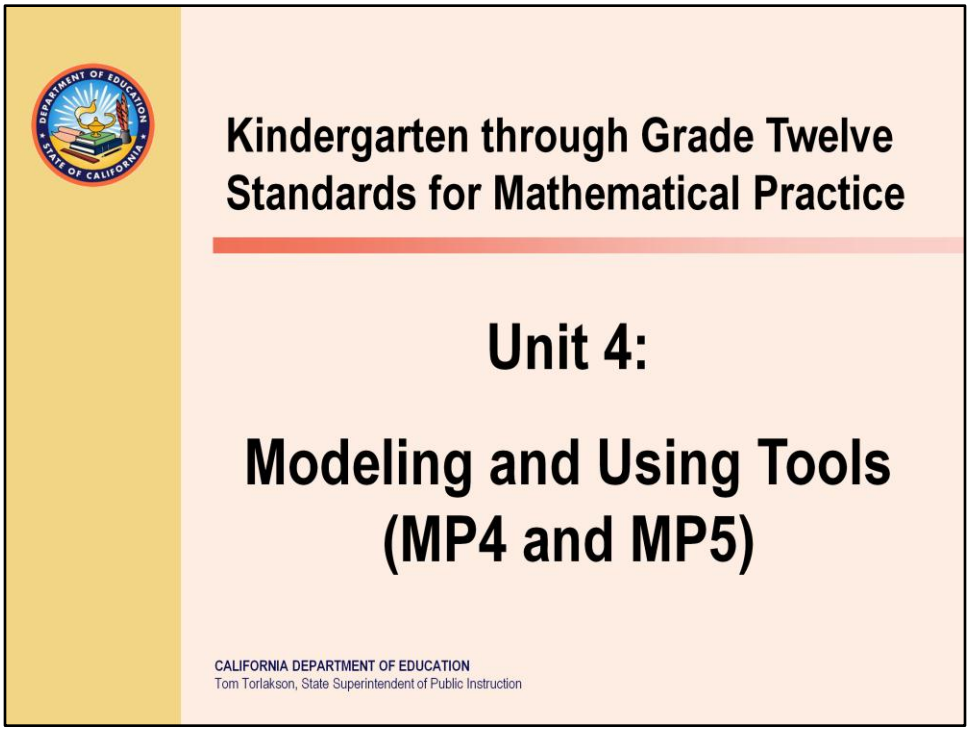
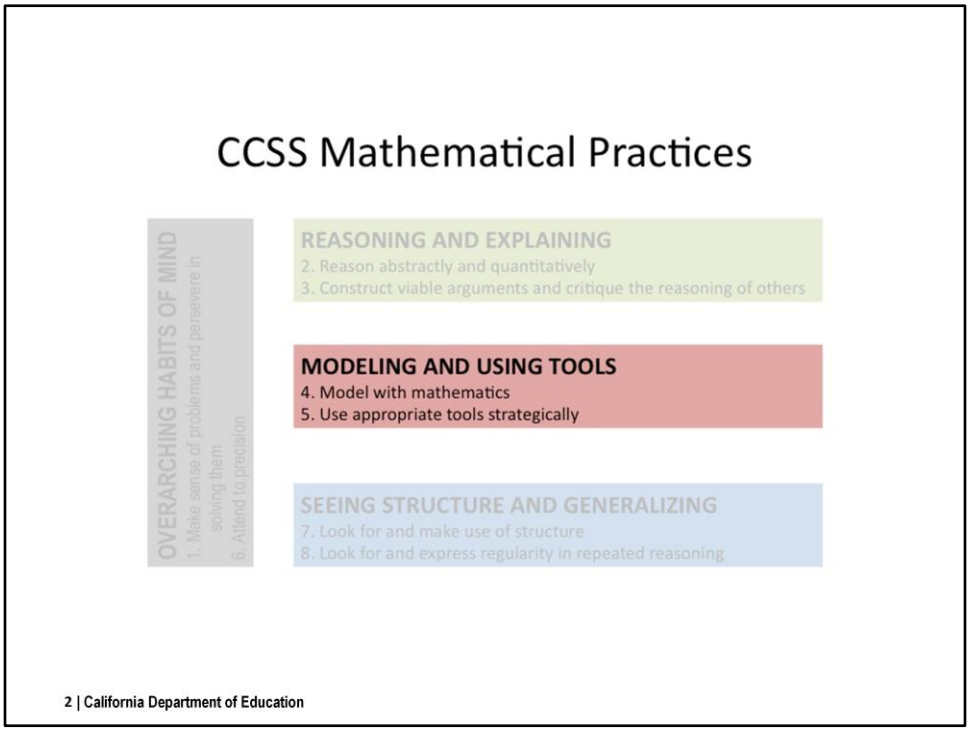


Talking Points:

Welcome to Unit 4.



Talking Points:

- In this unit, we will review the "Modeling and Using Tools" practices; MP4 and MP5.

Facilitator Note:

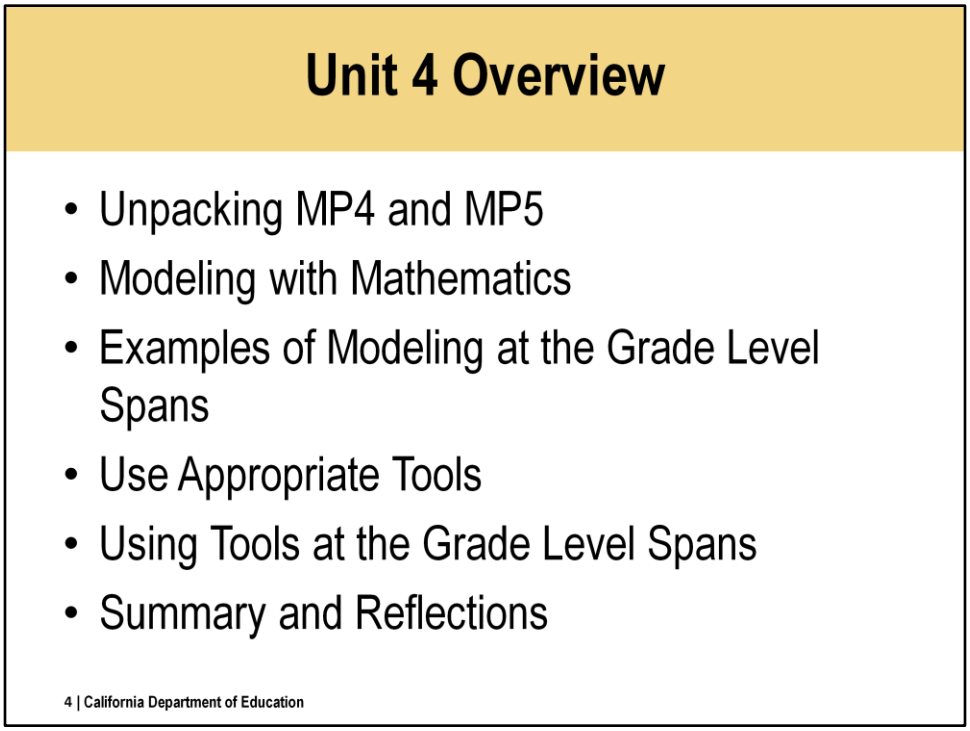
- Refer participants to the "CCSS Mathematical Practices" handout used in the previous section (**Handout 2.0**).

Unit 4 Learning Objectives

- You will be able to describe why, to be successful in mathematics, all students need to model and use tools.
- You will be able to explain what it means for students to model with mathematics.
- You will be able to explain what it means for students to use appropriate tools strategically.

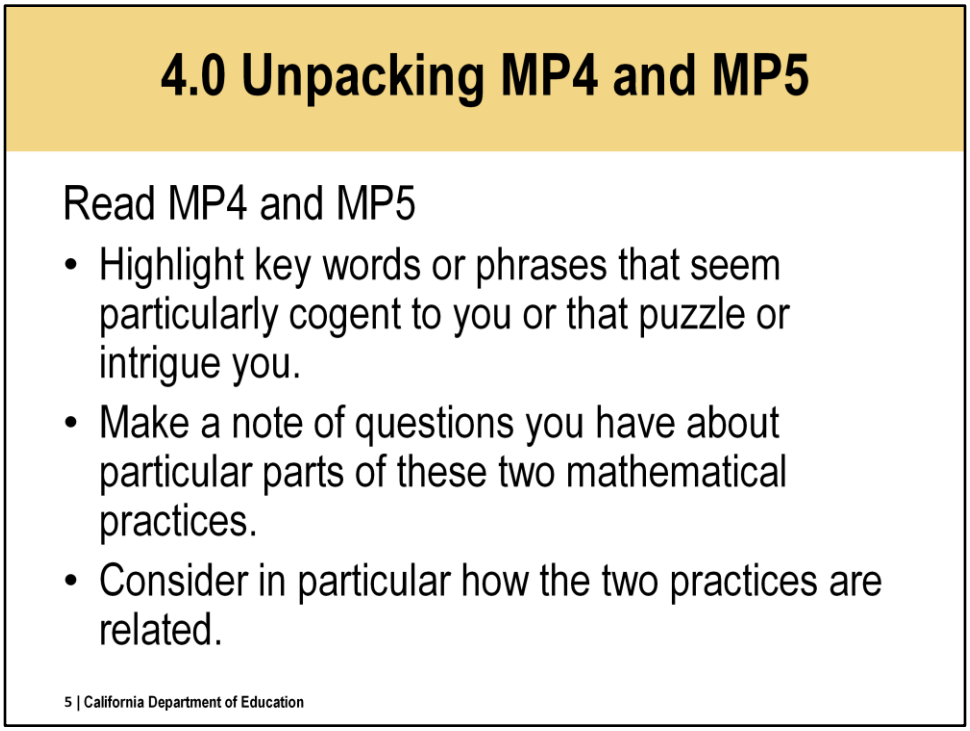
Talking Points:

- Unit 4 examines MP4 and MP5; the practices relating to modeling with mathematics and using tools. You will deepen your understanding of the various aspects of each standard as you work through this unit.
- By the end of this unit…[review bullets on slide]



Talking Points:

- This unit contains the following sections: [Review slide]



Facilitator Notes:

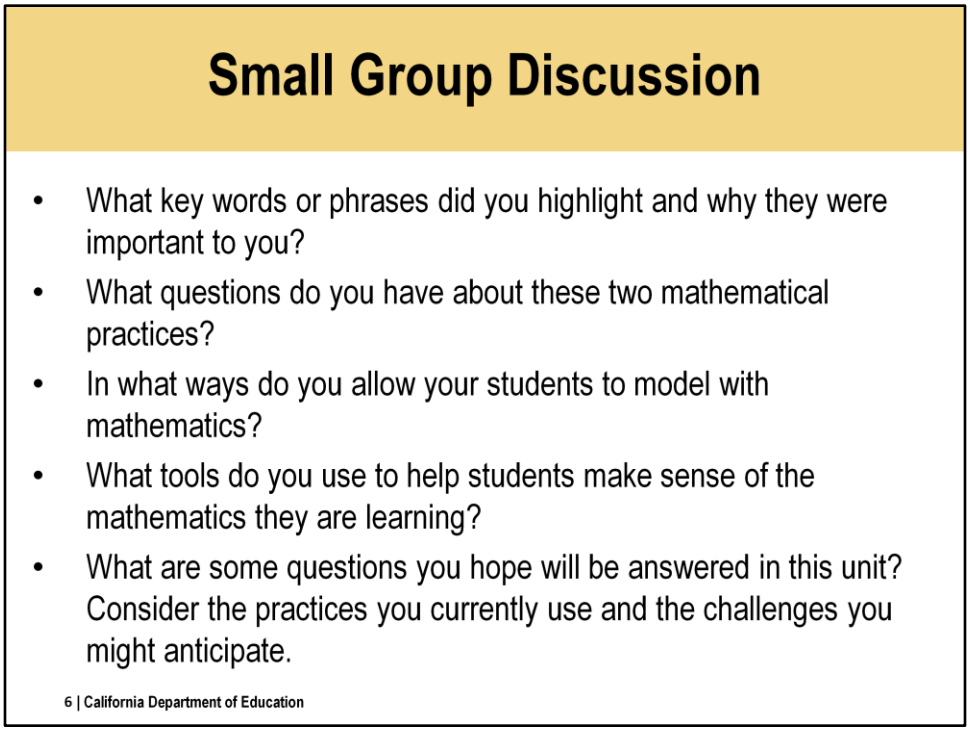
- Allow approximately 15 minutes to complete this activity.

Talking Points:

- Take a few minutes to review the modeling and using tools practices: "Unpacking MP 4 and MP 5" (**Handout 4.0.1**).
- As you read, note questions about particular parts of these two mathematical practices and highlight key words or phrases that puzzle, intrigue, or seem particularly cogent to you.
- Consider also how these two standards are related.

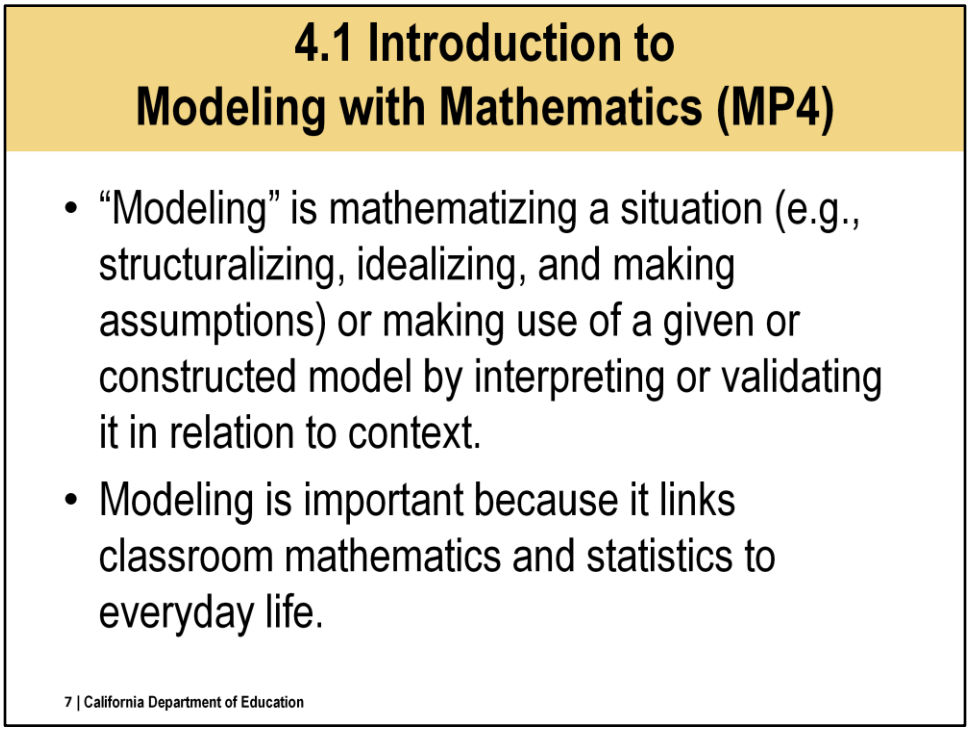
Facilitator Notes:

- Have participants share and chart questions they may have about MP4 and MP5.
- Have participants compare and contrast MP4 and MP5.



Facilitator Notes:

- After participants have discussed questions in small groups, lead a whole group discussion on the responses.



Facilitator Notes:

- Review slide.
- Have participants discuss what they think modeling looks like at their grade level.

The Program for International Student Assessment (PISA) Rubric

Continuum of Modeling Problems:

- **Level 0:** Purely computational or context is unnecessary for solving
- **Level 1:** Directly translatable from a context (e.g., a simple word problem from which students can formulate an equation).
- **Level 2:** A model can be modified to satisfy changed conditions. Allow students to study patterns and relationship between quantities, and represent these patterns and relationships using words, numbers, symbols, and pictures. May have multiple solution strategies, but usually only one correct solution.
- **Level 3:** Have no predetermined solution. Require students to make assumptions about the context, develop strategies to solve them, check answers, present results, and possibly revise solution strategies and begin the process over again .

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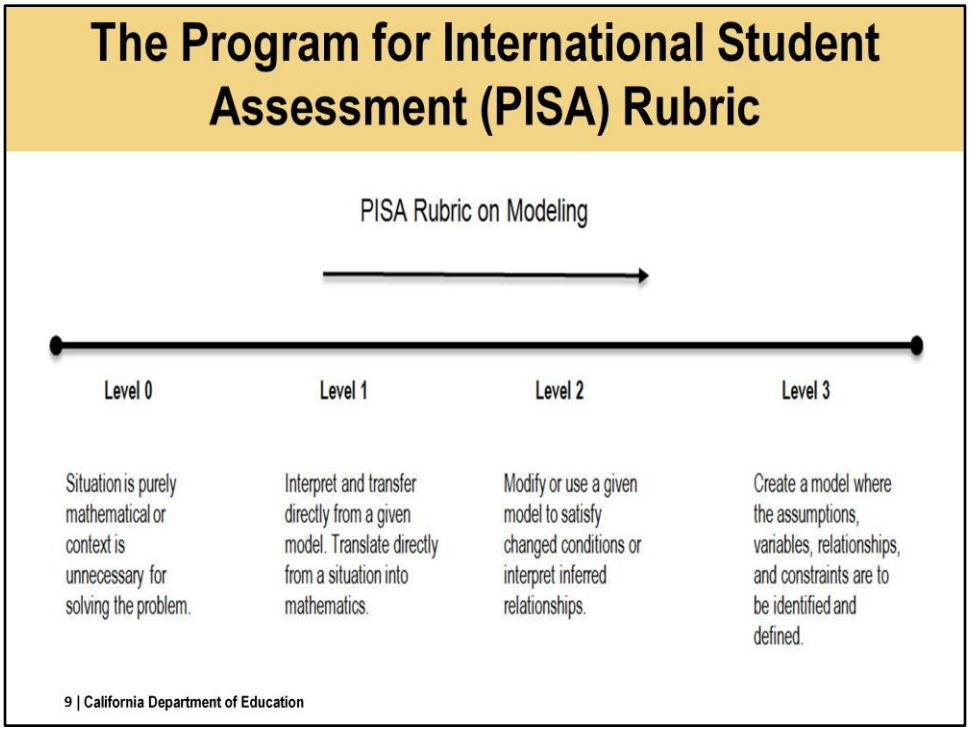
Talking Points:

- Modeling within mathematics is a broad standard that includes within its definition a wide range of mathematical problems.
- The (PISA) rubric defines modeling problems on a continuum of four different levels:
  - **Level 0:** Problems which are purely computational or context is unnecessary for solving them.
  - **Level 1:** Problems which are directly translatable from a context. An example is a simple word problem from which students can formulate an equation.
  - **Level 2:** Problems where a model can be modified to satisfy changed conditions. Such problems allow students to study patterns and relationship between quantities, and represent these patterns and relationships using words, numbers, symbols, and pictures. These can be problems that have multiple solution strategies, but usually have only one correct solution.
  - **Level 3:** Problems which have no predetermined solution. Such problems require students to make assumptions about the context, develop strategies to solve them, check their answers, present results, and possibly revise their solution strategies and begin the process over again .

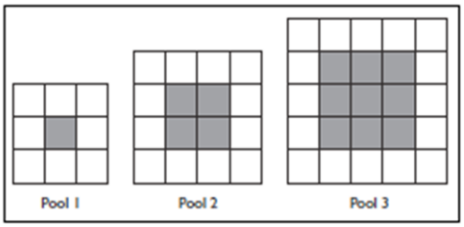


Talking Points:

- The ability to model with mathematics should grow over time as a student progresses through school.
- By the time a student reaches high school, they should be engaging in level 3 modeling.
- This will require students to engage in level 1 modeling in the lower grades, and level 2 modeling in the middle grades.

An Example of Modeling with Mathematics

Tat Ming is designing square swimming pools. Each pool has a square center that is the area of the water. Tat Ming uses blue tiles to represent the water. Around each pool there is a border of white tiles. Below are pictures of the three smallest pools that he can design with the blue tiles for the interior and white tiles for the border (Ferrini-Mundy, et al., 1997).

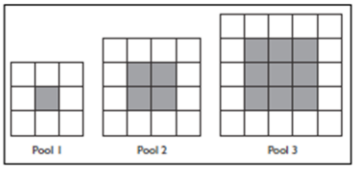


Talking Points:

- The next activity will include grade span specific questions for the "Tiling the Pool Problem."
- This task will allow you to experience modeling with mathematics appropriate for the grade level you teach.
- As it is written, this task represents a level 2 modeling problem.

[review slide]

Tiling Pool Problem



- Complete the "Tiling Pool Problem" at your grade level span.
- Discuss the mathematics that emerge with other members of your grade span group.

Facilitator Notes:

- Refer participants to the "Tiling the Pool Problem" **(Handout 4.1.1)**.
- Divide the group into grade spans.

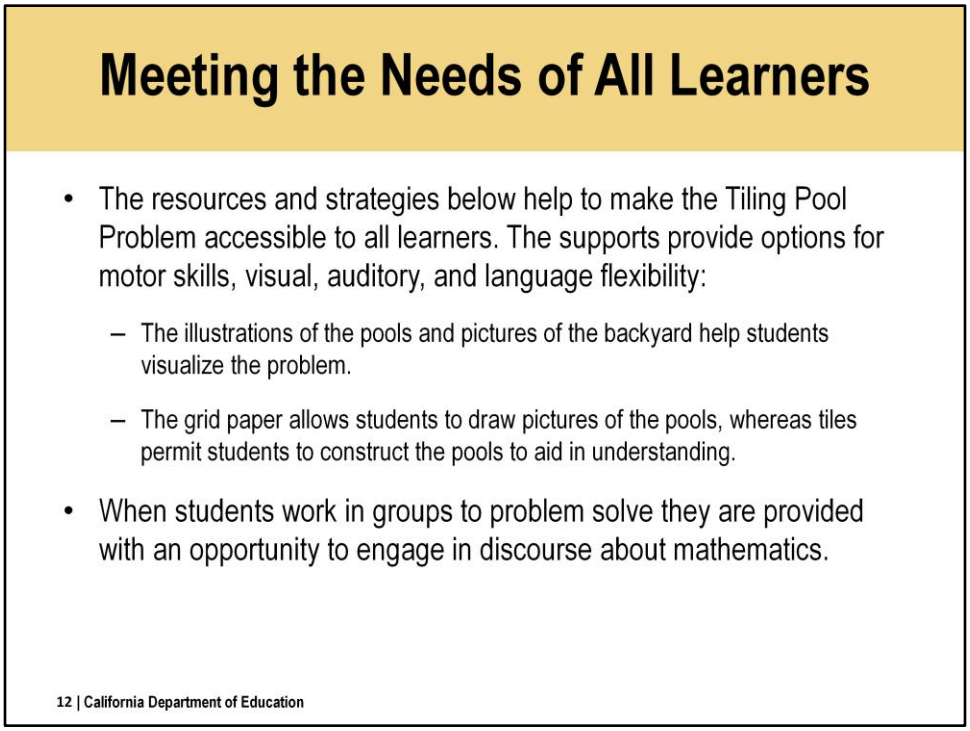
[review slide]

- Have participants work on their grade span problem and discuss the mathematics that emerge from the problems.
- If there is more than one grade span group, have each share out their solutions.

Talking Points:

[after completing the problem]

- List ways that the Tiling Pool Problem meets the needs of all learners.
- Share your list with a partner.



Talking Points:

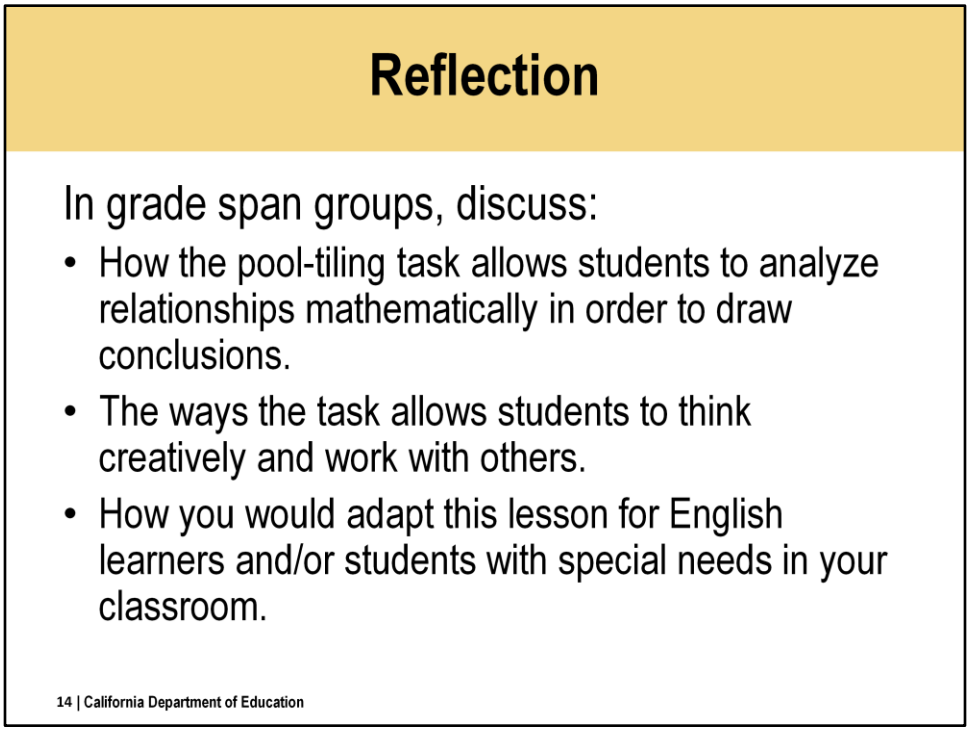
- The resources and strategies shown on the slide help make the Tiling Pool Problem accessible to all learners, especially English learners and students with special needs.
- The supports provide options for motor skills, visual, auditory, and language flexibility.
  - The illustrations of the pools and pictures of the backyard help students visualize the problem.
  - The grid paper allows students to draw pictures of the pools, whereas tiles permit students to construct the pools to aid in understanding.
- When students work in groups to problem solve they are provided with an opportunity to engage in discourse about mathematics.

4.1 Summary

- Modeling is mathematizing a situation, or making use of a model by interpreting it in relation to the context.
- Modeling is important because it links mathematics to everyday life.
- Modeling with mathematics exists along a continuum;
  - from problems that are directly translatable from a context,
  - to problems that have no predetermined solutions and require students to make assumptions, develop solution strategies, present results, and possibly revise their strategies.

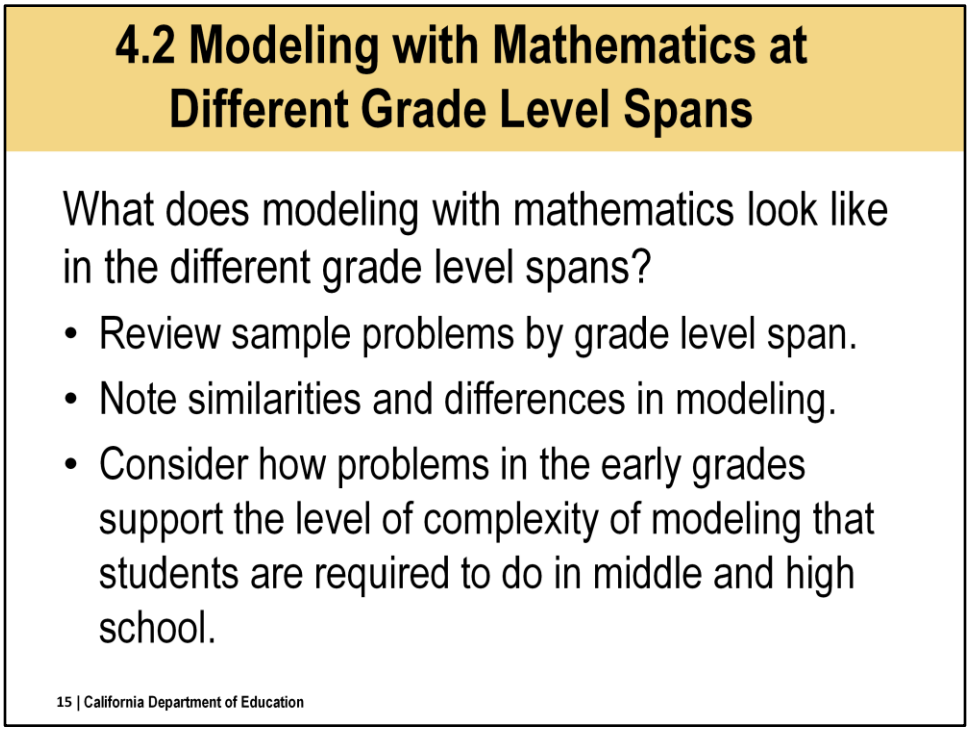
Facilitator Notes:

- Discuss each bullet point.



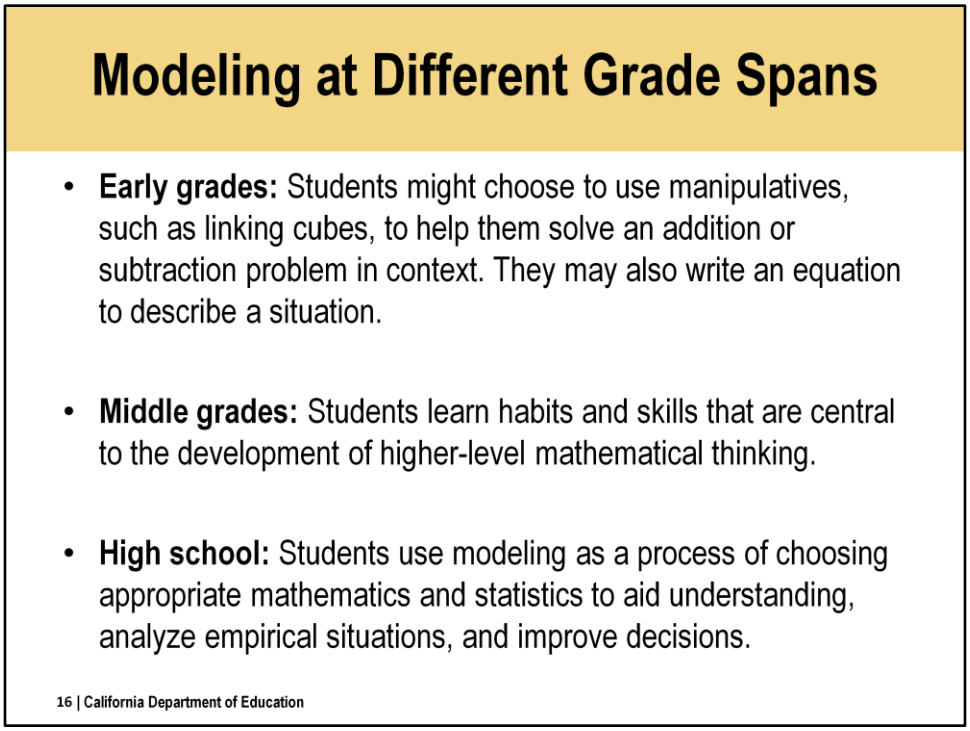
Facilitator Notes:

- Divide the participants into grade level span groups.
- Have them discuss the questions.
- Have each group share their discussion.



Talking Points:

- In this section, we will review sample problems by grade level span and note similarities and differences in modeling.
- As we review these problems, consider how problems in the early grades support the level of complexity of modeling that students are required to do in middle and high school.

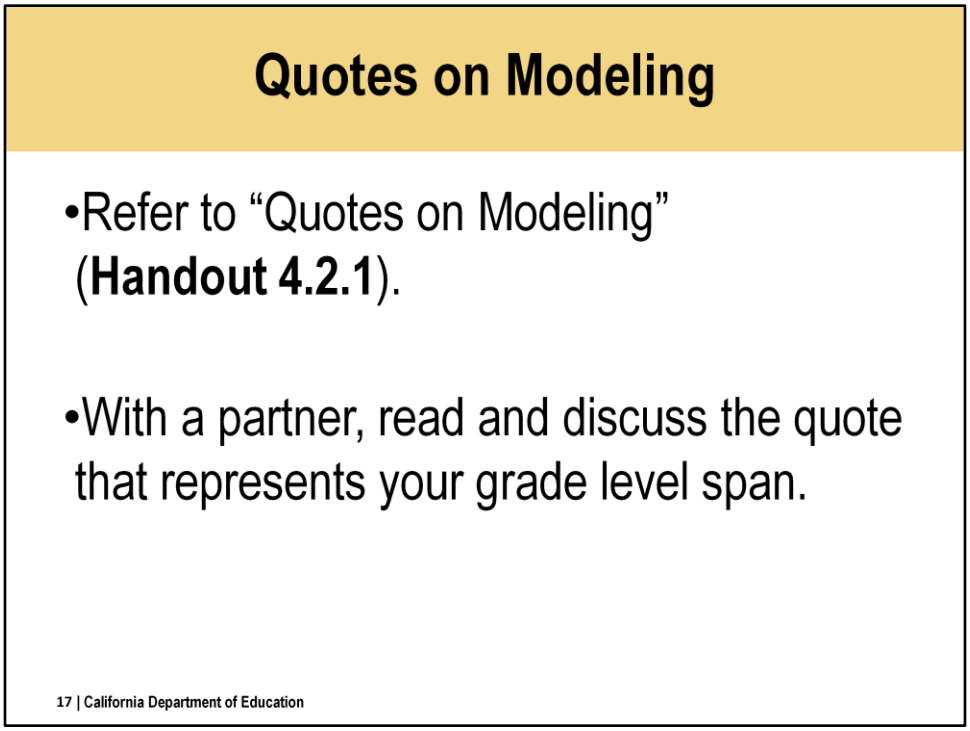


Talking Points:

- In the early grades, students might choose to use manipulatives, such as linking cubes, to help them solve an addition or subtraction problem in context. They may also write an equation to describe a situation.
- In the middle grades, students learn habits and skills that are central to the development of higher-level mathematical thinking.
- By high school, students use modeling as a process of choosing appropriate mathematics and statistics to aid understanding, analyze empirical situations, and improve decisions.

Facilitator Note:

- Ensure participants are in grade level spans for the next several activities.



Facilitator Note:

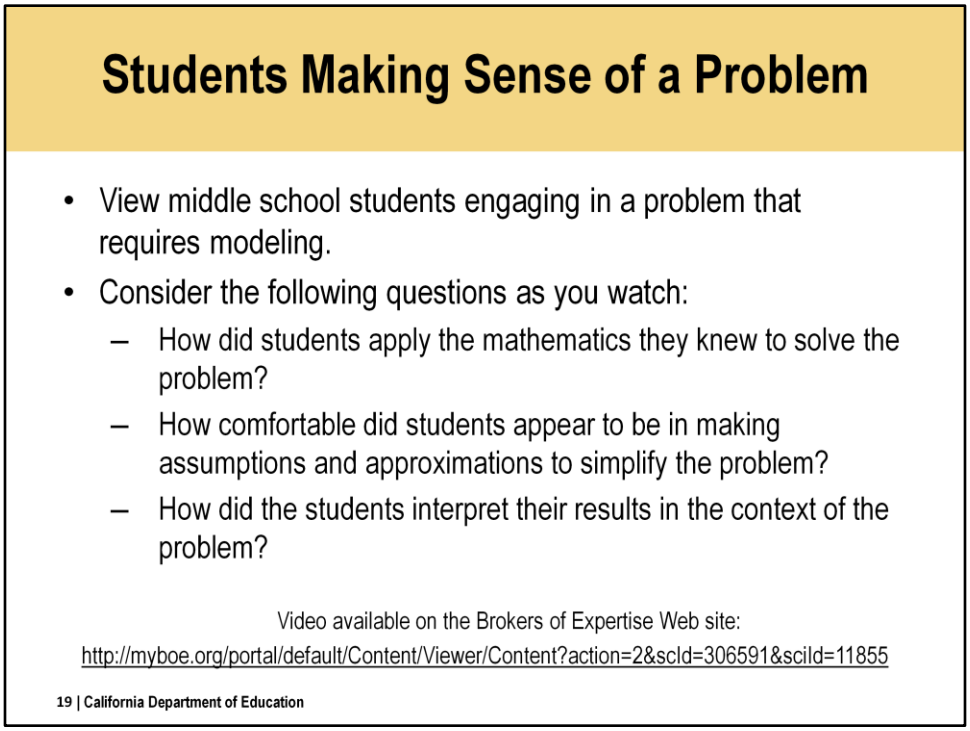
- Ensure that participants are sitting in grade level span groups.
- Refer participants to "Quotes on Modeling" (**Handout 4.2.1**).
- Have participants read and discuss the quote that represents their grade level span in pairs.

Examples of Modeling Activities

- **Refer** to the "Examples of Modeling Activities" handout that represents your grade level span (**Handouts 4.2.2 – 4.2.4)**.
- **Review** the problems, noting the level of each as defined on the PISA rubric.
- **Complete** the problems and compare solutions and strategies.
- **Discuss** how the difference in the subtle wording of the problems change the difficulty level.

Facilitator Notes:

- Review slide.



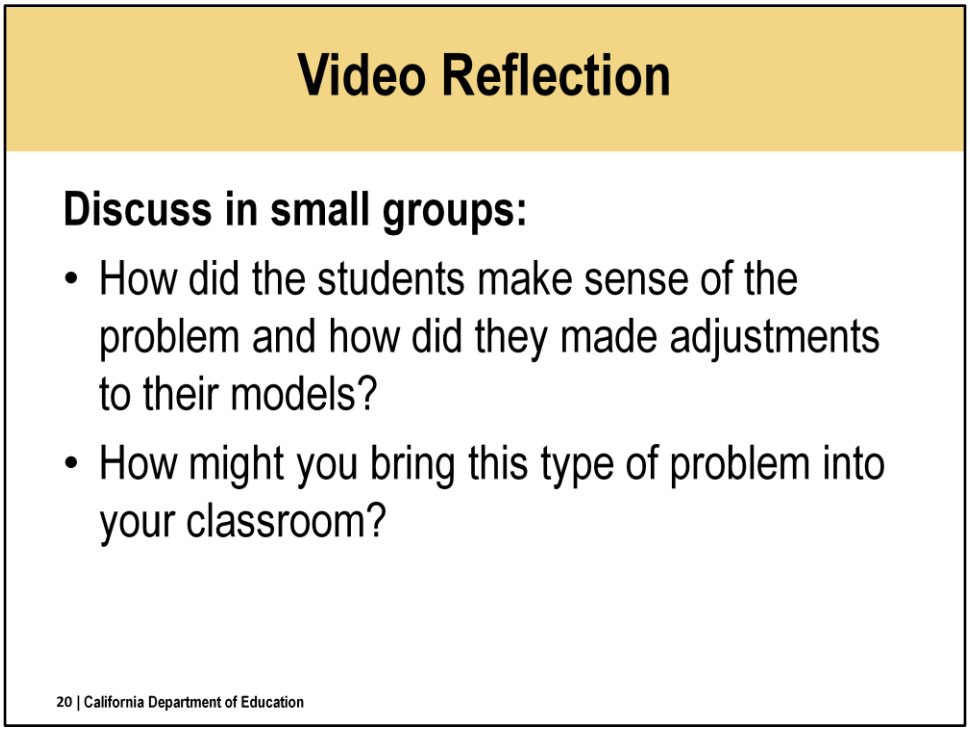
Facilitator Notes:

Video length: 6 minutes. Prepare Internet video hyperlink: http://myboe.org/portal/default/Content/Viewer/Content?action=2&scId=306591&sciId=11855

Talking Points:

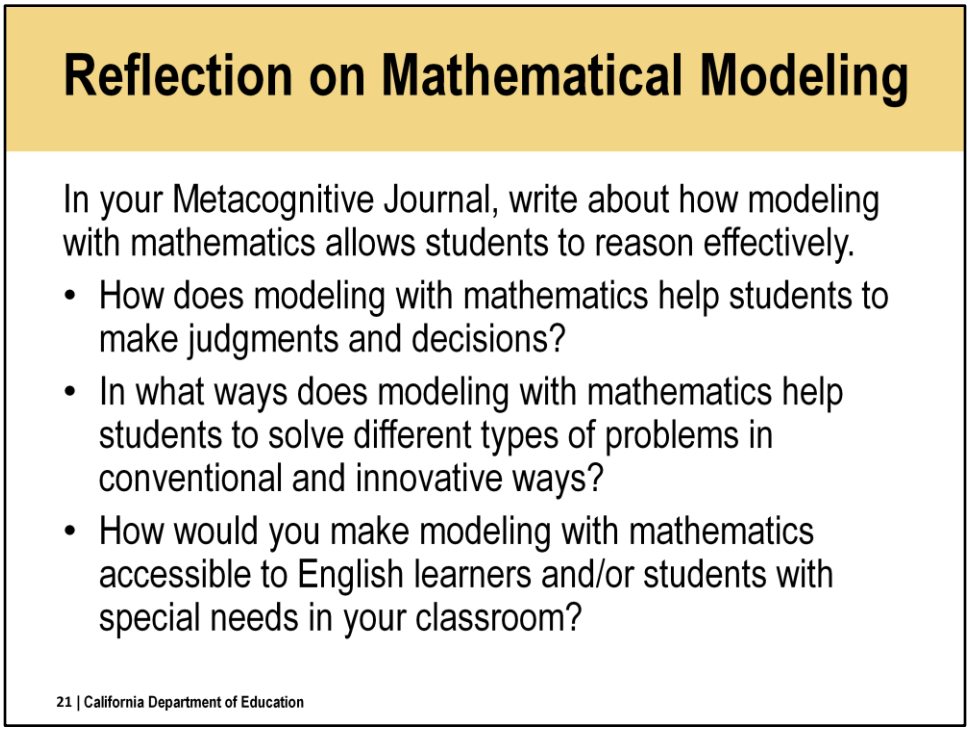
- View the video of a middle school teacher engaging her students in a problem that requires them to model with mathematics.
- Consider the questions on this slide as you watch.

[Have participants discuss questions with a partner after viewing the video]



Facilitator Notes:

- Have participants discuss questions from previous slide before discussing the two topics on this slide.
- Have each group share key comments.



Facilitator Note:

- Have participants respond to each prompt in their metacognitive journals.

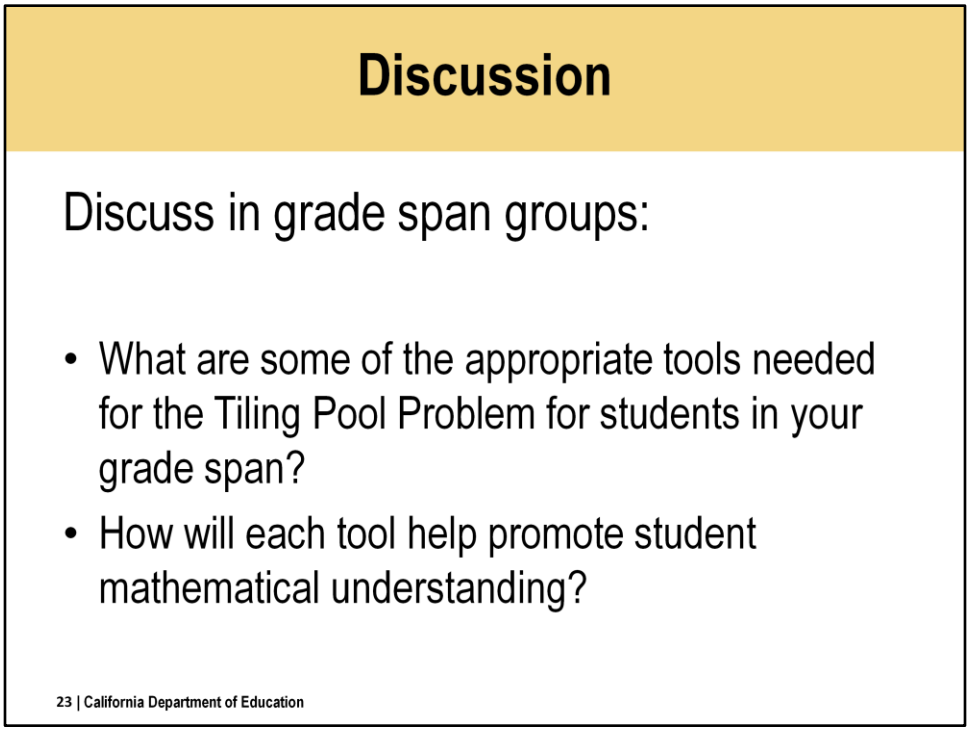
4.3 Introduction to Using Appropriate Tools Strategically (MP5)

When modeling with mathematics, the use of appropriate tools is essential for students to acquire the most out of a mathematical task.

- Tools are any object, picture, drawing, or figure that represents a mathematical concept used by students to make meaning of the mathematics, or on which the relationship for that mathematical concept can be imposed (Van De Walle, 2004).
- Tools allow students to interactively reason and engage in discourse about mathematics, helping them to construct mathematical meanings and explain their ideas.

Facilitator Notes:

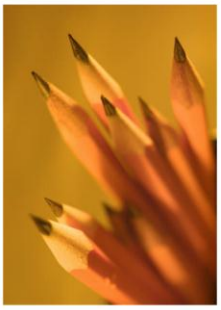
- Review slide.
- Ask participants what they believe it means to use tools appropriately.

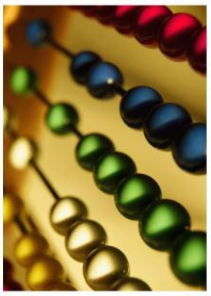


Talking Points:

- Refer to the Tiling the Pool Problems from Unit 4.1.
- Discuss the questions on the slide.
- List the tools needed on chart paper.
- Share with the whole group and discuss the different tools across the grade spans.

Tools for Each Grade Span





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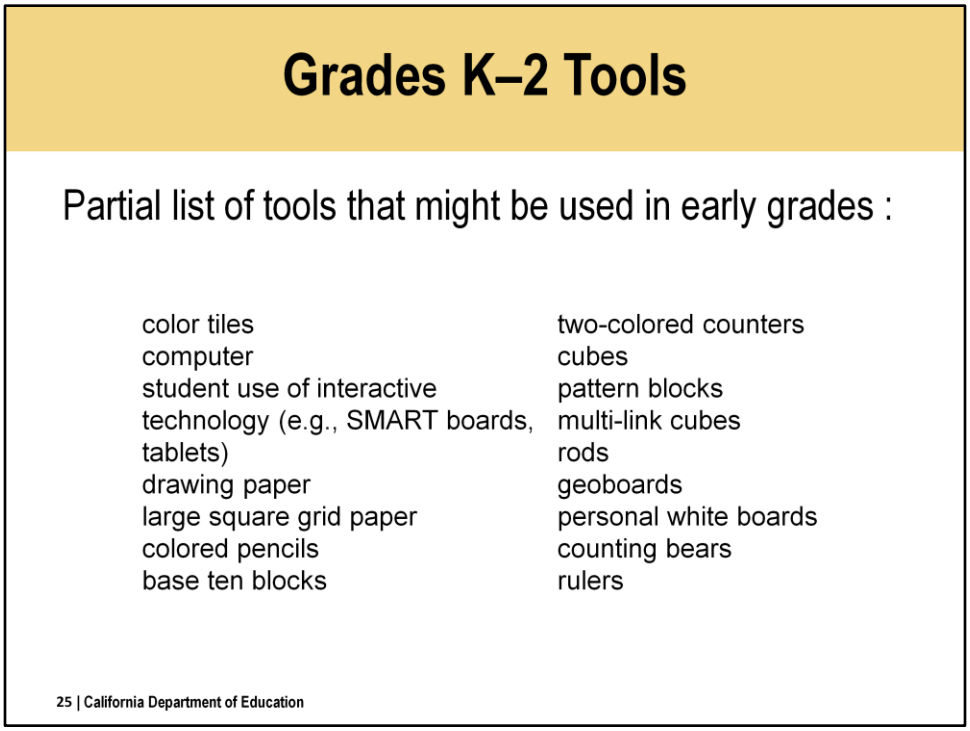
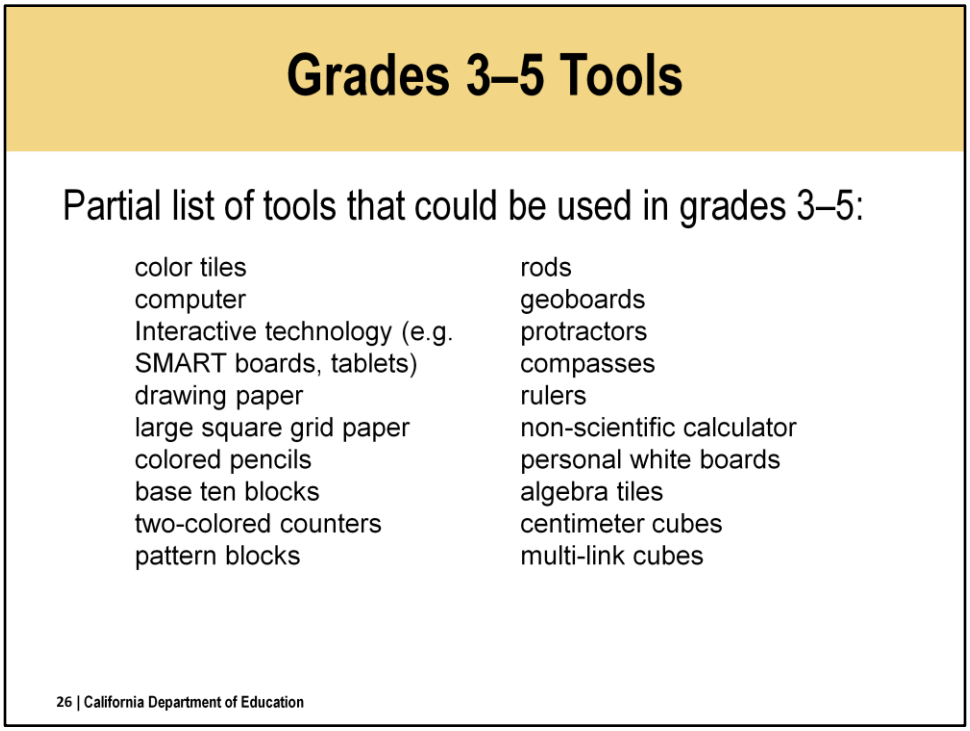
Facilitator Notes:

- Ensure participants are in grade span groups, K–2, 3–5, and 6–12.
- Refer participants to the appropriate quote for each grade level (**Handout 4.3.1**).
- Ask participants to write down all of the tools they use in their grade levels.
- Share the lists on the following slides.



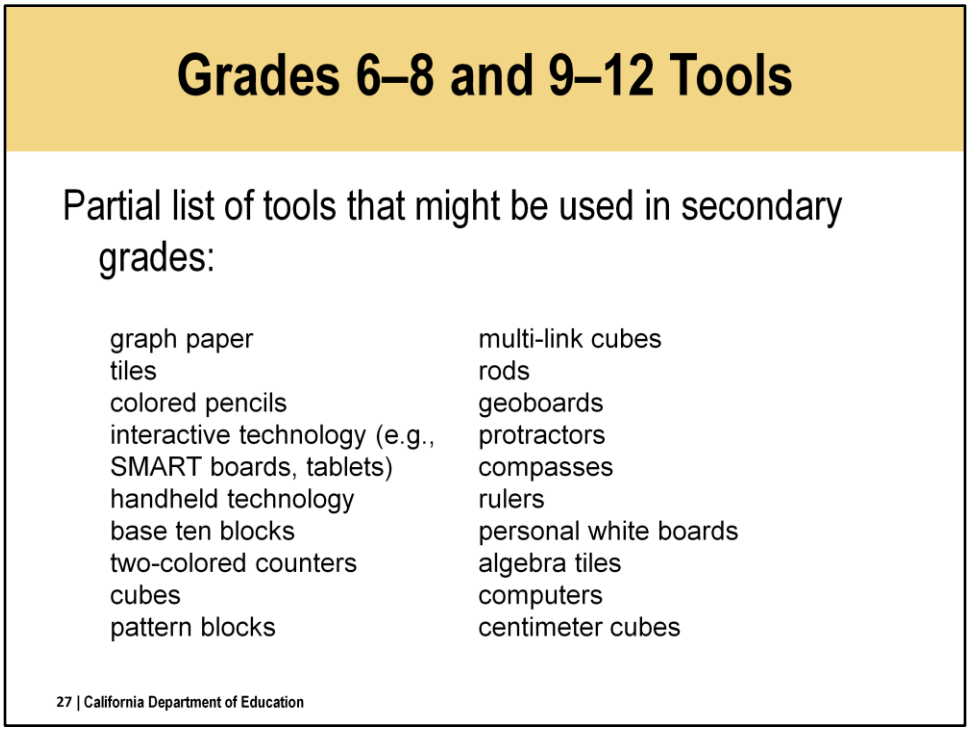
Talking Points:

- A partial list of tools that might be used in the early grades includes: [refer to slide]
- K–2 participants, please compare your lists with this one.



Talking Points:

- A partial list of tools that might be used in grades 3–5 includes: [refer to slide]
- Grades 3–5 participants, please compare your lists with this one.



Talking Points:

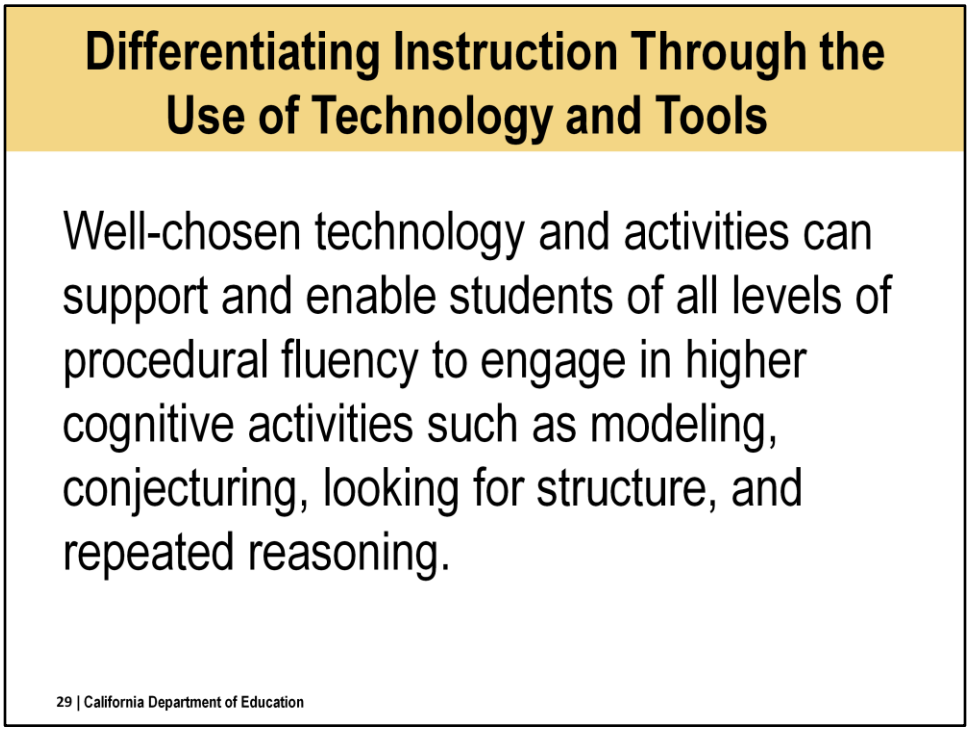
- A partial list of tools that might be used in the secondary grades includes: [refer to slide]
- Secondary participants, please compare your lists with this one.

Differentiating Instruction Through the Use of Technology and Tools

- The National Council of Teachers of Mathematics (NCTM) encourages the use of technology as a tool to support and extend mathematical reasoning and sense making, gain access to mathematical content and problem-solving contexts, and enhance computational fluency (NCTM, 2000).
- Carefully selected technology tools have the potential to provide multiple opportunities for differentiated instruction to support both struggling and advanced students.

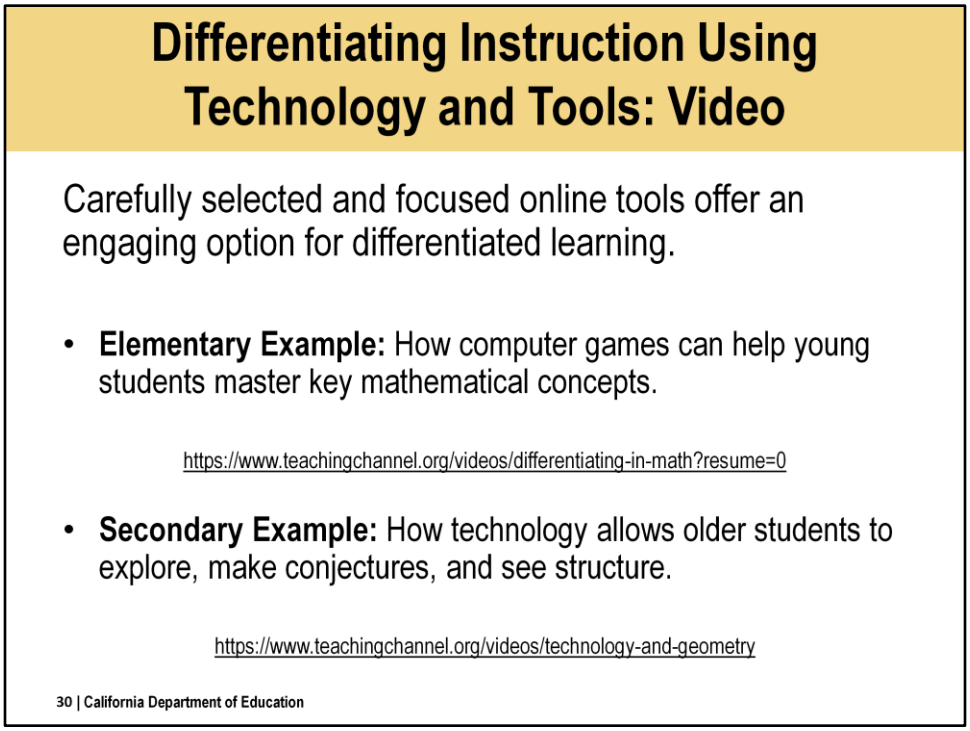
Facilitator Notes:

- Review slide.
- Have participants discuss how technology and tools help differentiate student learning.



Talking Points:

- When looking for geometric or algebraic structures it is useful to be able to check and examine many cases and examples.
- When these cases need to be done by hand, it can exclude many students who do not have the procedural speed and accuracy from engaging in the SMP.
- Well-chosen technology and activities can support and enable students of all levels of procedural fluency to engage in higher cognitive activities such as modeling, conjecturing, looking for structure, and repeated reasoning.
- Following are some video examples of a wide range of students using technology as a tool to engage in cognitively demanding mathematics.



Facilitator Notes:

Video length: Approximately 16 minutes for two videos. Prepare Internet video hyperlinks:

https://www.teachingchannel.org/videos/differentiating-in-math?resume=0

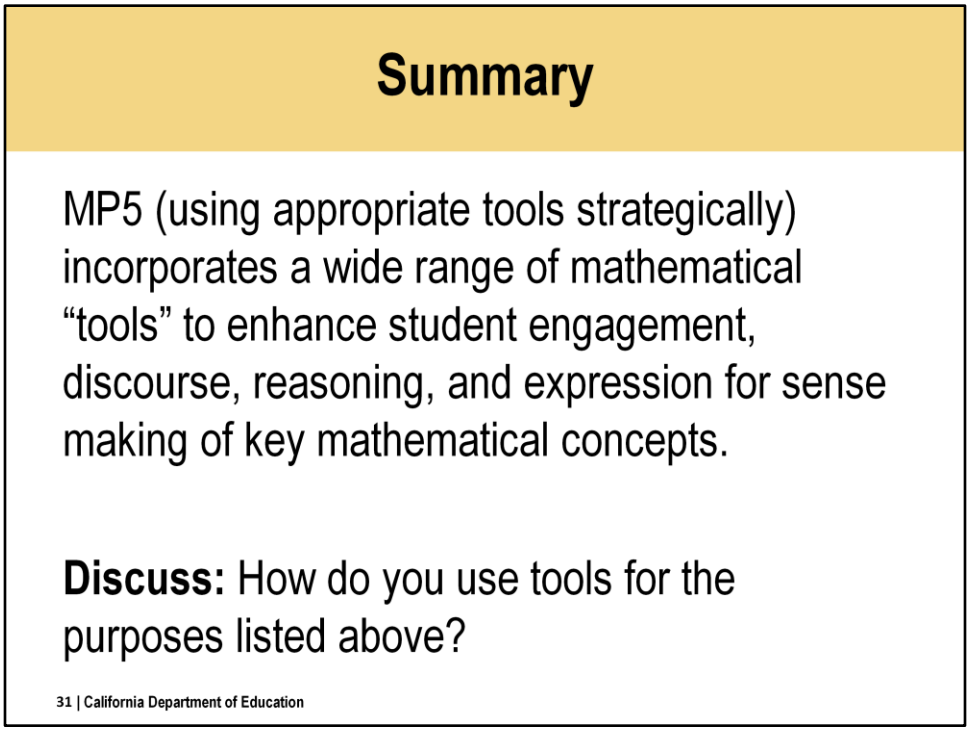
https://www.teachingchannel.org/videos/technology-and-geometry

Talking Points:

- If selected carefully and focused on specific mathematical objectives, online tools offer an engaging option for differentiated learning.
- The first video shows an example of how computer games can help young students master key mathematical concepts.
- The second video shows an example of how technology can benefit older students, especially those who have entered high school with gaps in their mathematical skills. Notice how technology allows students to explore, make conjectures, and see structure.

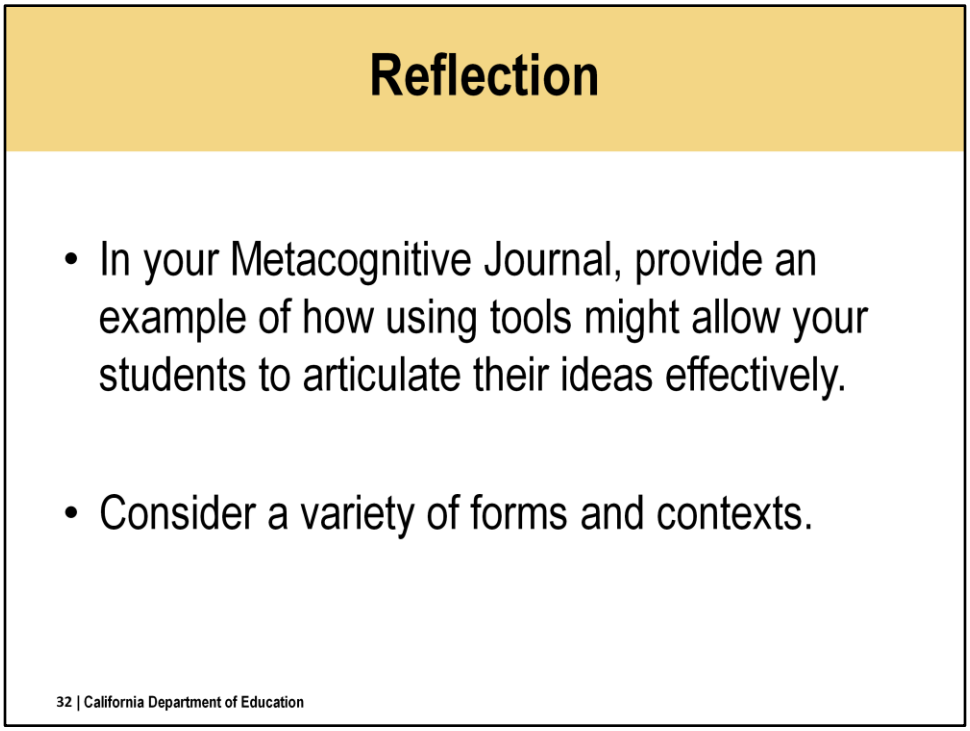
Note: There are two clips in the second video that are relevant to this unit:

- First Clip: 05:30-7:30 - Students use dynamic geometry software to explore conjectures in geometry.
- Second Clip: 18:35-26:50 - Students use handheld technology to facilitate their understanding of quadratic functions.



Facilitator Notes:

- Review slide.
- Ask participants to discuss how they use tools for the purposes listed on the slide.



Facilitator Notes:

- Have participants record their ideas in their metacognitive journals on how tools help their students articulate ideas effectively.

4.4 Use of Tools at Different Grade Level Spans

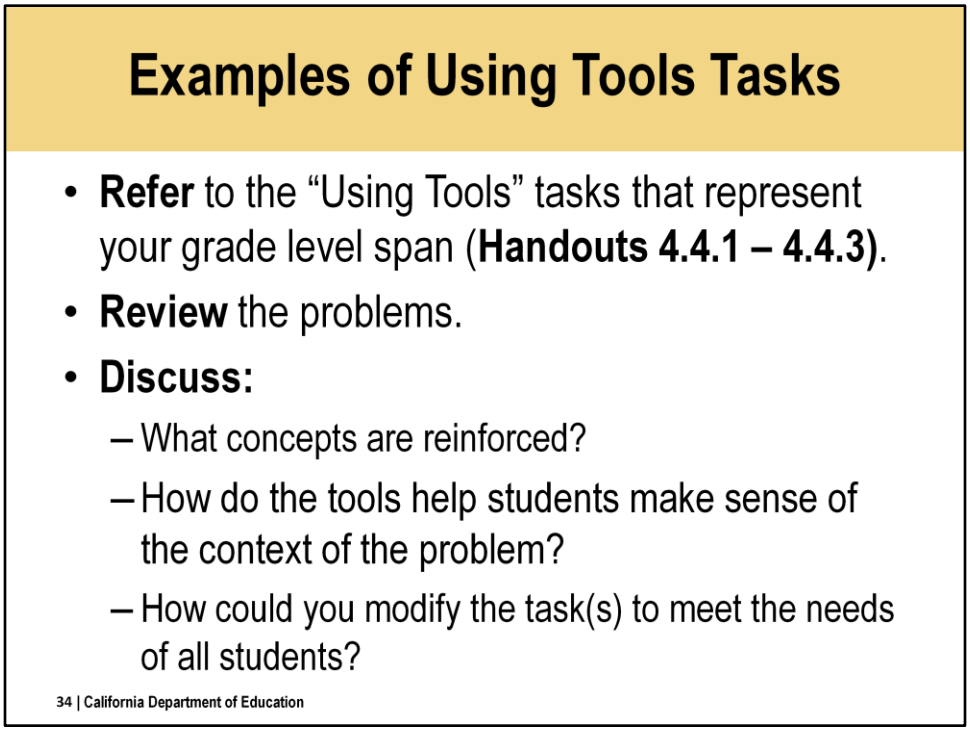
The use of tools is important at all grade level spans, but the types of tools and how they are used can differ depending on grade level.

Discuss:

- What tools are universal for all grade levels and what tools differ by grade level spans?
- What characterizes the difference in the appropriateness of a tool at a particular grade level?

Facilitator Notes:

Review slide and facilitate discussion.



Talking Points:

- Refer to your grade level span in the "Using Tools" tasks (**Handouts 4.4.1 – 4.4.3)** for examples of how students can use tools to solve a task.
- Discuss with your grade span group what concepts are reinforced through the activities.

Facilitator Notes:

- If time permits, have participants play the game/complete the tasks.
- Ask participants what modifications they might make for their students.

Using Tools Strategically: Video Examples

- The following video examples demonstrate the appropriate and strategic use of tools.
- As you view the video(s), take note of how students use tools to solve problems that are posed.

Elementary Example:

https://www.teachingchannel.org/videos/teaching-fractions

Secondary Example:

https://www.teachingchannel.org/videos/teaching-subtracting-integers

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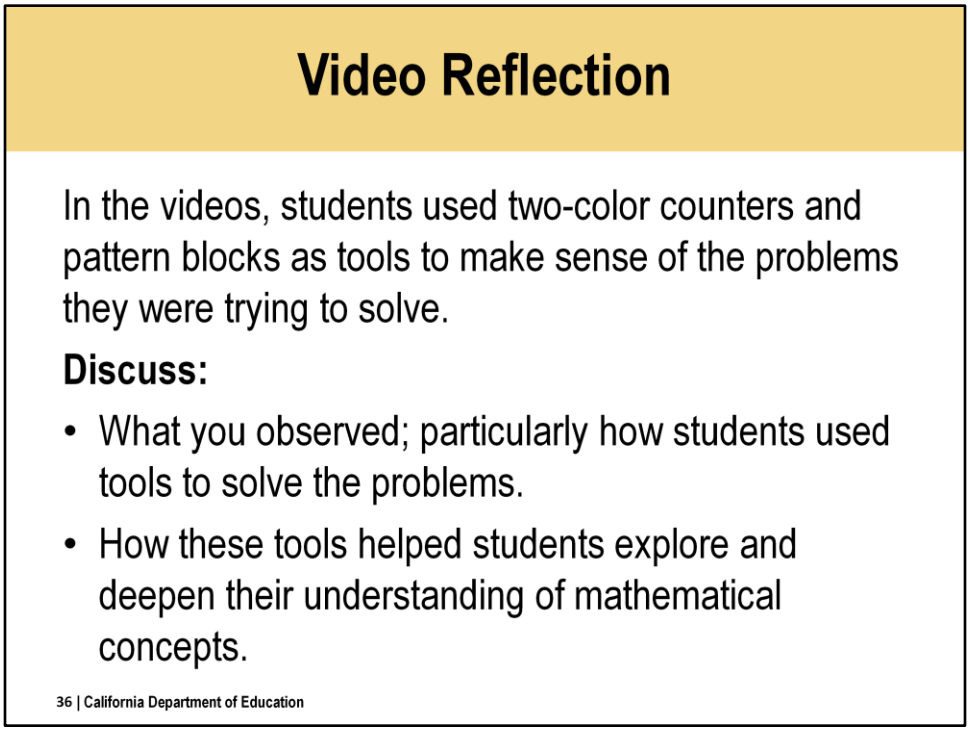
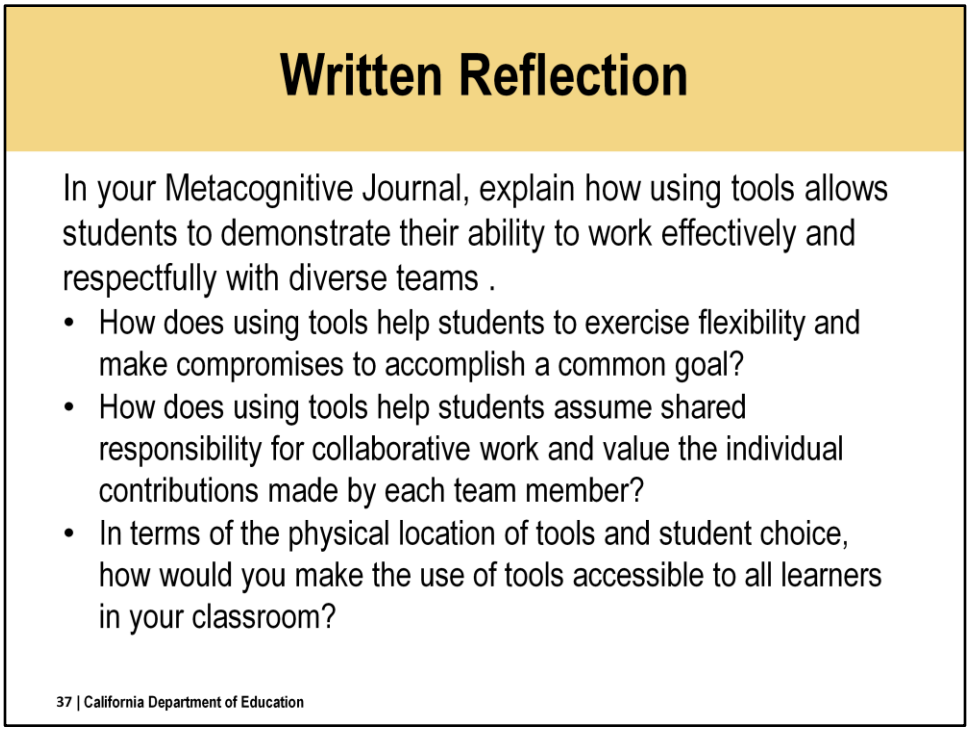
Facilitator Notes:

- If technology is available, have participants watch only the video that represents their grade span.
- If viewing videos by grade span is impractical, show all videos to the whole group and then have participants discuss the video that applies to their grade span in grade level groups.



Facilitator Notes:

- Ensure participants are grouped into elementary and secondary grade spans.
- Have them discuss the how students used tools to solve problems and how these tools helped students explore and deepen their understanding of mathematical concepts.



Facilitator Notes:

- Have participants respond to the prompts on the slide in their Metacognitive Journals.
- Invite participants to share their ideas with the whole group.

4.4 Summary

- There are tasks at all grade levels where tools are an important part of the problem solving process, whether the task is a modeling problem or computational in context.
- Sometimes it is clear what the tool should be for a task, and sometimes students can make choices about the tools that will work best for them.
- Providing students with a choice of tools differentiates for the variety of learners in the classroom, including English learners and students with special needs.

Facilitator Notes:

- Review slide.
- Ask participants what tools they would like to try in their classrooms that they have never used before and why.

Unit Summary and Reflection

- In this unit, you have considered the Modeling and Using Tools practices; MP4 and MP5.
  - Modeling with mathematics requires students to make assumptions and approximations to simplify a situation, realizing these may need revision later.
  - Modeling with mathematics requires students to interpret mathematical results in the context of the situation and reflect on whether they make sense.
  - Using tools strategically requires that students are familiar with appropriate tools to decide when each tool is helpful; knowing both the benefits and limitations.
  - Using tools strategically requires students to detect possible errors, identify relevant external mathematical resources, and use them to pose or solve problems.

Facilitator Notes:

- Review slide bullets with participants.
- Ask participants to add additional ideas and thoughts that surfaced as they completed the activities.

Unit Reflection

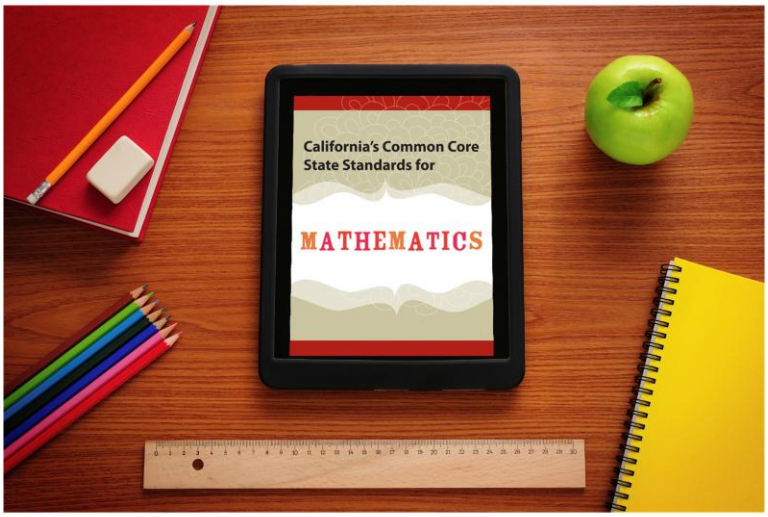
Think about your understanding of these two practices and how they work together. Refer to the four PISA modeling levels and respond to the following questions:

- For your grade level, describe a task for each of the three levels of modeling. Provide justification for why the task fits that particular level.
- Name three tools that you currently have access to in your classroom and describe a task for which each tool would be appropriate.
- Students need to know the procedures for using manipulatives to learn mathematics. Create a plan to reach this goal in your classroom.
- Although the use of calculators is often debated, describe an appropriate task in which a calculator would be used.

Facilitator Notes:

- In grade level groups, respond to the prompt.
- Ask participants to think about how they will engage students in modeling that is appropriate to their grade level.
- Invite participants to share ideas to the whole group.

California's Common Core State Standards for Mathematics



(TRANSITION SLIDE)