Rectangle-Diamond Method for Factoring Trinomials: Why It Works

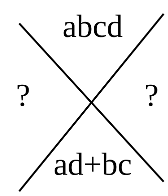
(courtesy of Dr. Bruce Jackson)

How do you factor a trinomial in the form $ax^2 + bx + c$ where a>1?
The rectangle-diamond method works because the rectangle helps visualize the original multiplication of binomials in a mathematically valid way that highlights its internal relationships, namely that the cross-products of the rectangle have to be equal. Begin with the basic multiplication $(ax + b)(cx + d)$ and place the partial products into the rectangle:

	ax	$+ b$
cx	acx^2	bcx
$+ d$	adx	bd

Clearly, both products of diagonally opposite cells have to contain x^2 plus all 4 constants: $abcdx^2$. The coefficients in our original problem are actually the result of multiplying four different constants, a, b, c, and d, and the nature of the problem would be more visible if it were expressed as $acx^2 + adx + bcx + bd$ or better yet: $acx^2 + (ad + bc)x + bd$. Then it's clear that the coefficient of the middle term is actually the sum of two numbers ad and bc that must themselves be factors of the product abcd. The "diamond" is a handy way to pick the factors of abcd that yield the correct middle term of the trinomial.

	ax	$+ b$
cx	acx^2	?
$+ d$	?	bd



So, for the trinomial $6x^2$! x ! 15 it helps greatly to realize that !90 is the number abcd and hence that 9x and !10x will occupy the empty corners of the rectangle above. From there it's very easy to determine what a, b, c, and d must be to give this result:

	$3x$	! 5
$2x$	$6x^2$	!10x
$+ 3$	$9x$	!15

(3x ! 5)(2x + 3)