



Common Core State Standards Professional Learning Module Series

Mathematics:

Kindergarten through Grade Twelve Standards for Mathematical Practice

CALIFORNIA DEPARTMENT OF EDUCATION
Tom Torlakson, State Superintendent of Public Instruction

Welcome to the Series

Common Core State Standards (CCSS) Professional Learning Modules

Available on the Brokers of Expertise Web Site at www.myboe.org

Welcome Message

Lupita Alcala, Deputy Superintendent
Instruction and Learning Support Branch
California Department of Education (CDE)

<http://myboe.org/portal/default/Group/Viewer/GroupView?action=2&qid=2996>

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Introduction and Overview Standards for Mathematical Practice

- Module Goals
- Overview of the CCSS for Mathematics
- Introduction to Module Units
- Metacognition and Reflection
- Pre-Assessment

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Module Goals

The overarching goals of the “Mathematics: Kindergarten through Grade Twelve (K–12) Standards for Mathematical Practice” module are to enable educators to:

- Deepen their understanding of the Common Core Mathematical Practice (MP) standards
- Support the learning of all students

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The CCSS Initiative

Supports states in developing tools to support teaching and learning:

- Curricula
- Instructional materials
- Assessments
- Professional learning opportunities

The CCSS are:

- Internationally benchmarked
- Focused on readiness for college and career

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The CCSS for Mathematics

Two Types of Mathematics Standards:

- **Standards for Mathematical Content:** Define what students should understand and be able to do at each grade level.
- **Standards for Mathematical Practice (SMP):** Ensure that the “processes and proficiencies of mathematics” are at the heart of teaching and learning.

These two types of standards are interconnected; interacting to improve instruction and learning for all students.

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Introduction to SMP Module Units

- Unit 1: Teaching and Learning the SMP
- Unit 2: Overarching Habits of Mind (MP1 and MP6)
- Unit 3: Reasoning and Explaining (MP2 and MP3)
- Unit 4: Modeling and Using Tools (MP4 and MP5)
- Unit 5: Seeing Structure and Generalizing (MP7 and MP8)
- Unit 6: Summary, Next Steps, and Resources

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Metacognition and Reflection

Metacognition: Thinking about thinking, reflecting on one's personal thought processes, allowing the learner to connect new knowledge to prior knowledge, formulate ideas, and clarify thinking.

- You will have the opportunity to predict, reflect, make conjectures, and synthesize through:
 - entries in your Metacognitive Journal
 - small and large group discussion

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Pre-Assessment

Assess your knowledge of the SMP prior to beginning the lessons:

- Complete the “Pre-Assessment” (**Handout 1.0**)
- Work independently, without discussion or assistance from others

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Unit 1

Teaching and Learning the Standards for Mathematical Practice

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Unit 1: Learning Objectives

Upon completion of this unit:

- You will be able to describe the difference between the Standards for Mathematical Content and the SMP.
- You will be introduced to the eight SMP and understand their importance and connection to the content standards.
- You will be able to explain how the SMP define what it means for a student to be mathematically proficient.

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1.0 Observing Students

Observe students engaged in mathematics:

Grade 2: Classroom Discussion: Describing Patterns

Grade 6: Classroom Discussion: Sorting Angles

Grade 8: Classroom Discussion: Speed and Rate Using Technology

As you watch the video(s), think about the following:

- What questions are being asked and at what level?
- What strategies are being used?

Videos available on the Brokers of Expertise Web site at
<http://myboe.org/portal/default/Content/Viewer/Content?action=2&scld=306591&scild=11776>

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Observing Students, Cont.

Discuss in small groups:

- What were the students doing?
- In what ways were they engaged?
- Describe your observations of students' thinking, problem solving, and interactions.

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1.1 Content Standards: Important Shifts

The content standards are organized around domains and coherent clusters. They seek to illustrate increased:

- **Focus:** Strongly emphasized at appropriate grade levels for deep understanding
- **Coherence:** Thinking across grade levels, and linking to major topics within grade levels
- **Rigor:** Pursue, with equal intensity, conceptual understanding; procedural skill and fluency; and application in major topics

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Content Standards: Balance and Depth of Understanding

K–8 Content Standards by Domains

1	2	3	4	5	6	7	8
Measurement and Data				Statistics and Probability			
Number and Operations in Base Ten				The Number System			
Operations and Algebraic Thinking				Expressions and Equations			
Counting and Cardinality			Number and Operations—Fractions		Ratios and Proportional Relationships		Functions

- Examples of tasks can be viewed at the standards level in each grade.

Source: Illustrative Mathematics: <http://illustrativemathematics.org>

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Balance and Depth of Understanding, Cont.

High School Conceptual Categories

NUMBER AND QUANTITY	ALGEBRA	FUNCTIONS	GEOMETRY	STATISTICS AND PROBABILITY
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- The Modeling Conceptual Category (not represented in the chart) includes standards in Higher Mathematics Courses and is embedded throughout other Conceptual Categories indicated by a (?) symbol.
- Examples of tasks can be viewed at the standards level in the conceptual categories.

Source: Illustrative Mathematics <http://illustrativemathematics.org>

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Additional Information

- “Mathematics: Kindergarten Through Grade Eight Learning Progressions” professional learning module:
 - Brokers of Expertise <http://myboe.org/>
- Tools for the Common Core Standards: <http://commoncoretools.me/category/progressions/>

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1.2 The Practice Standards

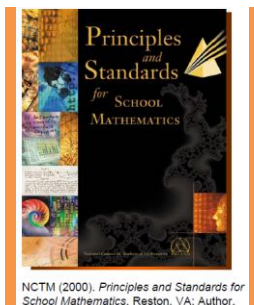
The mathematical practices “rest on important ‘processes and proficiencies’ with longstanding importance in mathematics education.

Source: CCSS for Mathematics, p.6

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Processes:

National Council of Teachers of Mathematics (NCTM)

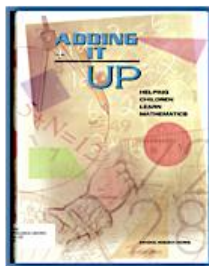


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- Problem Solving
- Reasoning and Proof
- Communication
- Representation
- Connections

Proficiencies:

National Research Council



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- Adaptive Reasoning
- Strategic Competence
- Conceptual Understanding
- Procedural Fluency
- Productive Disposition

Doing and Using Mathematics

The SMP are about doing and using mathematics to foster reasoning and sense-making.



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Ongoing Work

- The practice standards are not intended to be a list of processes and proficiencies to be checked off for mastery or completion.
- Student performance may range from novice to expert based on the content, the activities, and the task in which the student is engaged.

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The Eight SMP

- MP1: Make sense of problems and persevere in solving them**
- MP2: Reason abstractly and quantitatively**
- MP3: Construct viable arguments and critique the reasoning of others**
- MP4: Model with mathematics**

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The Eight SMP, cont.

- MP5. Use appropriate tools strategically**
- MP6. Attend to precision**
- MP7. Look for and make use of structure**
- MP8. Look for and express regularity in repeated reasoning**

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For All Learners, All the Time

- Each practice standard begins with the three words, “**mathematically proficient students.**”
- Proficiency is a goal for ALL students.
- The practice standards are for all students at all times, regardless of their abilities.

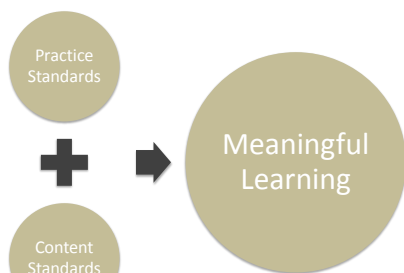
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Self-Reflection Survey

How comfortable and confident are you in supporting all students to be successful in the eight mathematical practices?

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1.3 Interaction of Practice Standards and Content Standards



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Meaningful Learning

Meaningful learning requires:

- Access to a coherent and connected curriculum
- An environment that promotes discourse, reflection, collaboration, and use of appropriate tools
- Doing mathematical problems and tasks which necessitate problem solving, reasoning, and sense-making

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Focus and Coherence

The video, “Mathematical Practices, Focus and Coherence” provides an overview of the interrelationship of the mathematics content and practice standards.

- Features Jason Zimba, one of the lead writers of the CCSS for Mathematics

Video available on the Council of Chief State School Officers
Web site at

http://programs.ccsso.org/ccv/JZ_4.m4v

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Time to Talk

- What are some of your initial observations about the eight SMP?
- What are some commonalities of the SMP across the grade levels?
- What are some ways in which you might connect content standards with the SMP in your classroom?

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Bringing Structure

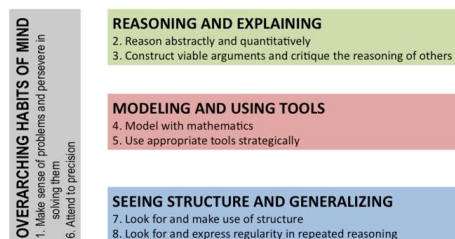
“If you think about it long enough you can associate just about any practice standard with any content standard, but this sort of matrix thinking can lead to a dilution of the force of the practice standards — if you try to do everything all the time you end up doing nothing.”

Bill McCallum, March 10, 2011

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The Structure

CCSS Mathematical Practices

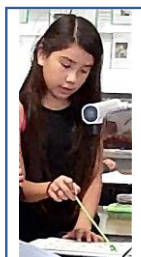


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Connecting Content and Practice

Points of Intersection:

- The SMP do not exist in isolation; doing and using mathematics involves connecting content and practice.
- The content standards are based on procedure and on understanding — both are stressed equally.



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Impact of Understanding

“Students who lack understanding of a topic may rely on procedures too heavily.... In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.”

Source: CCSS for Mathematics

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Importance of Understanding

“Without understanding, a student may rely on procedures and may not represent problems coherently, justify conclusions, apply the mathematics to other situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an interview, or deviate from a known procedure to find a shortcut.”

Source: CCSS for Mathematics

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Possible Starting Points

Content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the SMP.

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An Example

Grade 4, Number and Operations — Fractions, Standard 3a:

Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

Source: CCSS for Mathematics

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Grain Size

“It is not expected that a single problem would engage students with all eight SMP; even a lesson may be too small to accomplish that.”



Source: Daro, 2010

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Organizing Mathematics for Learning

Over the course of such a larger segment of work, engagement of multiple standards should be evident rather than within a single lesson, task, or problem.

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Progress Check

So far, this unit has covered:

- Grouping practices to facilitate implementation
- Connecting practice and content standards
- Organizing mathematics

Write/Discuss: How might what you have learned so far impact your instructional practice?

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1.4 Meeting the Needs of ALL Students

The CCSS for Mathematics are for ALL students, including:

- Students receiving special education services
- English learners
- Underperforming students
- Gifted and talented students



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Language Learning and Mathematics

- Balance student activities that address both **conceptual** and **procedural** knowledge
- Use high cognitive demand tasks
- Instill beliefs that mathematics is sensible, worthwhile, and doable
- Provide opportunities for students to engage in all of the mathematical practices

Source: Moschkovich, 2012

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Students with Disabilities

The standards are “for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs.” (CCSS for Mathematics).

Accommodations include:

- Instructional supports for learning
- Instructional accommodation
- Assistive technology devices and services

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Students with Disabilities: Resources

- **Application to Students with Disabilities**
<http://www.corestandards.org/assets/application-to-students-with-disabilities.pdf>
- **Teaching Common Core Math Practices to Students with Disabilities**
<http://www.naset.org/3553.0.html>
- **The Special EDge**
http://www.calstat.org/publications/pdfs/Edge_summer_2012_newsletter.pdf
- **Teaching Mathematics CCSS to Students with Moderate to Severe Disabilities**
<http://cec.metapress.com/content/80146k52rkp56813/>
- **Six Principles for Principals to Consider in Implementing CCSS for Students with Disabilities**
<http://www.naesp.org/principal-septemberoctober-2012-common-core/access-common-core-all-0>

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Supporting English Learners: English Language Development Standards

“English Language Development Standards (ELD) for California Public Schools: Kindergarten through Grade Twelve” prioritizes **meaning and interacting** over language structure.

Language is emphasized as “a complex and social meaning-making resource to be fostered via intellectually challenging, interactive, and dialogue-rich contexts focused on content knowledge and linguistic development.”

ELD Standards Overview, p.18

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Understanding Language: Guiding Principles

1. Focus on students’ mathematical reasoning, not accuracy, in using language.
2. Shift to a focus on mathematical discourse practices; move away from simplified views of language.
3. Recognize and support students to engage with the complexity of language in mathematics classrooms.
4. Treat everyday language and experiences as resources, not as obstacles.
5. Uncover the mathematics in what students say and do.

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Teaching Mathematics for Conceptual Understanding for ELs



<http://ell.stanford.edu/publication/mathematics-common-core-and-language>

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21st Century Skills

Students must have an opportunity to develop the skills, processes, and knowledge that will prepare them for the challenges that they will face. 21st Century Skills include:

- Creativity and innovation
- Critical thinking and problem solving
- Collaboration
- Communication
- Construction and exploration of new understandings

Education Code Section 60207(d)

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21st Century Skills and the SMP

An example of interconnectedness (Source: P21 Common Core Toolkit)

21 st Century Skill	Standards for Mathematical Practice
Critical thinking and problem solving	<ul style="list-style-type: none"> Make sense of problems and persevere in solving them
	<ul style="list-style-type: none"> Reason abstractly and quantitatively
	<ul style="list-style-type: none"> Model with mathematics
	<ul style="list-style-type: none"> Look for and make use of structure

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Learning Objectives Revisited

- Describe the difference between the Standards for Mathematical Content and the SMP.
- Understand the importance of the eight standards for mathematical practice.
- Explain how the standards for mathematical practice define what it means for a student to be mathematically proficient.

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Talk about...

- Some of the new ideas you have gained in this introductory unit
- Questions you have
- Some of the challenges that you anticipate as you prepare to support your students in their use of the SMP

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Think about...

As you participate in the next four units, think about:

- How will you teach so that students may engage in the SMP?
- How will you assess the SMP?
- How will you support students as they deepen their mathematical understanding through engagement in the SMP?
- How will you provide opportunities for **all** students to engage in the SMP?

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Looking Ahead

As you work through Units 2–5, consider:

- Trying out some of the tasks in your classroom
- Sharing the ideas learned with your colleagues

Now onward to a greater understanding of the Standards for Mathematical Practice!

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California's Common Core State Standards for Mathematics

