Appendix A: A Classroom Vignette

This vignette is presented to ground the subsequent descriptions of the recommendations and to show how these recommendations play out in classroom interactions. The lesson excerpt presented below (Moschkovich, 1999) comes from a third-grade bilingual classroom in an urban California school. In this classroom, there were thirty-three students identified as Limited English Proficient. In general, this teacher introduced students to topics in Spanish and then later conducted lessons in English. The students had been working on a unit on two-dimensional geometric figures. For several weeks, instruction had included vocabulary such as “radius,” “diameter,” “congruent,” “hypotenuse,” and the names of different quadrilaterals in both Spanish and English. Students had been talking about shapes and the teacher had asked them to point, touch, and identify different shapes. The teacher identified this lesson as an English as a Second Language mathematics lesson, one where students would be using English in the context of folding and cutting to make Tangram pieces (see Figure 1).

Figure 1: A tangram puzzle

Vignette

1. Teacher: Today we are going to have a very special lesson in which you really gonna have to listen. You’re going to put on your best, best listening ears because I’m only going to speak in English. Nothing else. Only English. Let’s see how much we remembered from Monday. Hold up your rectangles . . . high as you can. (Students hold up rectangles) Good, now. Who can describe a rectangle? Eric, can you describe it [a rectangle]? Can you tell me about it?

2. Eric: A rectangle has . . . two . . . short sides, and two . . . long sides.

3. Teacher: Two short sides and two long sides. Can somebody tell me something else about this rectangle, if somebody didn’t know what it looked like, what, what . . . how would you say it.

4. Julian: Paralela [holding up a rectangle, voice trails off].

5. Teacher: It’s parallel. Very interesting word. Parallel. Wow! Pretty interesting word, isn’t it? Parallel. Can you describe what that is?

6. Julian: Never get together. They never get together [runs his finger over the top side of the rectangle].
Teacher: What never gets together?

Julian: The parallela . . . they . . . when they go, they go higher [runs two fingers parallel to each other first along the top and base of the rectangle and then continues along those lines], they never get together.

Antonio: Yeah!

Teacher: Very interesting. The rectangle then has sides that will never meet. Those sides will be parallel. Good work. Excellent work.

The vignette serves to show that English language learners can and do participate in discussions where they grapple with important mathematical content. Students were grappling not only with the definitions for quadrilaterals but also with the concept of parallelism. Student were engaged in mathematical communication because they were making claims, generalizing, imagining, hypothesizing, and predicting what will happen to two lines segments if they are extended indefinitely. To communicate about these mathematical ideas students used words, objects, gestures, and other students’ utterances as resources. This vignette also illustrates several instructional strategies that can be useful in supporting student participation in mathematical discussions. Some of these strategies are: asking for clarification, re-phrasing student statements, accepting and building on what students say, and probing what students mean. It is important to notice that this teacher did not focus directly on vocabulary development but instead on mathematical ideas and arguments as he interpreted, clarified, and rephrased what students were saying. This teacher provided opportunities for discussion by moving past student grammatical or vocabulary errors, listening to students, and trying to understand the mathematics in what students said. He kept the discussion mathematical by focusing on the mathematical content of what students said and did.

**Recommendation #1: Focus on Students’ Mathematical Reasoning, Not Accuracy in Using Language.**

*In the vignette:* Uncovering the mathematical content in Julian’s contributions is certainly a complex endeavor. Julian’s utterances in turns 4, 6, and 8 are difficult both to hear and interpret. He uttered the word “parallela” in a halting manner, sounding unsure of the choice of word or of its pronunciation. His voice trailed off, so it is difficult to tell whether he said “parallelo” or “parallela.” His pronunciation could be interpreted as a mixture of English and Spanish; the “ll” sound being pronounced in English and the addition of the “o” or “a” being pronounced in Spanish. The grammatical structure of the utterance in line 8 is intriguing. The apparently singular “parallela” is preceded by the word “the” which can be either plural or singular and then followed with a plural “when they go higher.” In any case, what is clear is that Julian made several attempts to communicate a mathematical idea in his second language. If we only focus only on his English proficiency, we would miss his mathematical reasoning. Julian is, in fact, accurately describing a property of parallel lines. This teacher moved past Julian’s unclear utterance and use of the term “parallela.” He focused on the mathematical content of what students said, not the mistakes they made. He attempted to uncover the mathematical content in what Julian had said. He did not correct Julian’s English, but instead asked questions to probe what the student meant.
Recommendation #2: Shift to a Focus on Mathematical Discourse Practices, Move Away from Simplified Views of Language.

In the vignette: What competencies in mathematical practices did Julian display? Julian was participating in three central mathematical practices: abstracting, generalizing, and imagining. He was describing an abstract property of parallel lines and making a generalization saying that parallel lines will never meet. He was also imagining what happens when the parallel sides of a rectangle are extended. If we only focused on vocabulary, we would miss Julian’s use of these important mathematical practices.

Recommendation #3: Recognize and Support Students to Engage with the Complexity of Language in Mathematics Classrooms.

In the vignette: What modes of expression did Julian and the teacher use? Julian used gestures and objects in his description, running his fingers along the parallel sides of a paper rectangle. The teacher also used gestures and visual displays of geometric figures on the blackboard. This example shows some of the complexity of language in the mathematics classroom.

Recommendation #4: Treat Everyday Language and Experiences as Resources, Not as Obstacles.

In the vignette: What language resources did Julian use to communicate his mathematical ideas? He used colloquial expressions such as “go higher” and “get together” rather than the formal terms “extended” or “meet.” These everyday expressions were not obstacles but resources.

Recommendation #5: Uncover the Mathematics in What Students Say and Do.

In the vignette: How did the teacher respond to Julian’s contributions? The teacher moved past Julian’s confusing uses of the word “parallela” to focus on the mathematical content of Julian’s contribution. He did not correct Julian’s English, but instead asked questions to probe what the student meant. This response is significant in that it represents a stance towards student contributions during mathematical discussion: listen to students and try to figure out what they are saying. When teaching English learners, this means moving beyond vocabulary, pronunciation, or grammatical errors to listen for the mathematical content in student contributions. (For a discussion of the tensions between these two, see Adler, 2001.)
In the vignette: What instructional strategies did the teacher use? The teacher used gestures and objects, such as the cardboard geometric shapes, to clarify what he meant. For example, he pointed to vertices and sides when speaking about these parts of a figure. Although using objects to clarify meanings is an important ESL instructional strategy, it is crucial to understand that these objects do not have meaning that is separate from language. Objects acquire meaning as students talk about them and these meanings are negotiated through talk. Although the teacher and the students had the geometric figures in front of them, and it seemed helpful to use the objects and gestures for clarification, students still needed to sort out what ‘parallelogram’ and ‘parallel’ meant by using language and negotiating common meanings for these words.

In the vignette: The teacher did not focus on vocabulary instruction but instead supported students’ participation in mathematical arguments by using three instructional strategies that focus more on mathematical discourse: 1) **Building on student responses**: The teacher accepted and built on student responses. For example in turns 4-5, the teacher accepted Julian’s response and probed what he meant by “parallel.” 2) **Asking for clarification**: The teacher prompted the students for clarification. For example, in turn 7 the teacher asked Julian to clarify what he meant by “they.” 3) **Re-phrasing**: The teacher re-phrased (or re-voiced) student statements, by interpreting and rephrasing what students said. For example, in turn 10 the teacher rephrased what Julian had said in turn 8. Julian’s “the parallela, they” became the teacher’s “sides” and Julian’s “they never get together” became “will never meet”. The teacher thus built on Julian’s everyday language as he re-voiced Julian’s contributions using more academic language.

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2. The question of whether mathematical ideas are as clear when expressed in colloquial terms as when expressed in more formal language is highly contested and not yet, by any means, settled. For a discussion of this issue, see Tim Rowland’s book *The Pragmatics of Mathematics Education: Vagueness in Mathematical Discourse*. 

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