Mathematics: Kindergarten through Grade 8
Learning Progressions
User’s Guide

California Department of Education
Sacramento, California
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Group Facilitator’s Guide: Preparation Notes

Background information

Prior to the workshop:

- Complete the online module (available on the Brokers of Expertise Web site at http://ccssplm.myboe.org).
- Review Standards and Point of Use Documents.

Please note:

- Key points, learning objectives, tasks with expected times and descriptions, suggestions to stimulate and advance learning, and presentation scripts are provided in the Notes section of the PowerPoint (PPT).
- The Teaching Channel videos are not embedded into the PPT. Hyperlinks to the videos are available on the PPT graphics as indicated below:
  - Slide 44: Zero Pairs, Manipulatives, and a Real-World Scenario https://www.teachingchannel.org/videos/teaching-subtracting-integers
- Prepare handouts, supplies, and equipment, and ensure Internet connectivity
- Read the “System Requirements” on pages 12 and 13 of the Independent User’s Guide within this document to ensure that your browser will support the videos.

Day of the Workshop:

- Set up equipment, prepare tables (materials and supplies), ensure Internet connectivity
- Establish norms when session begins (e.g., turn off cell phones or put in silent mode, refrain from texting, etc.).
Group Facilitator’s Guide: Preparation Notes

Materials, Supplies, and Equipment

Download from the Brokers of Expertise Web site and print in advance:

- PowerPoint Slides with Presenter Notes (pdf of PPT for facilitator use)
- Participant Packets (Presentation Slides) — one per person
- Participant Packets (Point of Use Documents) — one per person

“Point of Use” handouts are embedded in a pdf document. These are handouts needed for activities that appear at the point in which they are used.

- Slide 3: Pre-assessment
- Slide 12: Aspects of Mathematics Standards
- Slide 21: CCSS for Mathematics (excerpt from page 5)
- Slide 31: Learning Progressions, Part 2 Viewing Guide
- Slide 41: Completed Instructional Alignment Chart for Grades 6–8 (The Number System, Integers)
- Slide 42: Instructional Alignment Chart Template
- Slide 50: Academic Language Functions
- Slide 55: Supports for English Learners (Note Taking Guide included on page 2)
- Slide 56: Evidence of EL Strategies Chart
- Slide 58: Instructional Strategies for Students with Disabilities
- Slide 59–61: Universal Designs for Learning Principles
- Slide 67: Unit 1 Summary
- Slide 68: Unit 2 Summary
- Slide 69: Unit 3 Summary
- Slide 71: Post-assessment

HANDOUTS (in addition to the Participant Packets)— available on the Brokers of Expertise Web site:

- Slides 21, 26, 40, 42: California’s Common Core State Standards for Mathematics
- Slide 35: University of Arizona’s Progressions, K, Counting and Cardinality; K–5, Operations and Algebraic Thinking
Mathematics: K–8 Learning Progressions
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- Slide 35: University of Arizona’s Progressions, 6–8 Expressions and Equations
- Slide 51: Illustrative Mathematics Tasks
  - Grade K.OA.3, Christina’s Candies
  - Grade 1.OA.1, Boys and Girls, Variation 2
  - Grade 2.OA.3, Red and Blue Tiles
  - Grade 3.OA.3, Gifts From Grandma, Variation 1
  - Grade 4.OA.1–3, Comparing Growth, Variation 1
  - Grade 5.OA.2, Video Games Scores
  - Grade 6.EE.2, Distance to School
  - Grade 7.EE.1, Writing Expressions
  - Grade 8.EE.7, The Sign of Solutions

VIDEOS:
- Slide 8: The Mathematics Standards: How They Were Developed and Who Was Involved (8:11 minutes)
- Slide 12: The Mathematics Standards and the Shifts They Require (1:14 minutes)
- Slide 17: The Importance of Mathematical Progressions from the Student Perspective (3:04 minutes)
- Slide 22: Learning Progressions, Part I (8:11 minutes)
- Slide 28: The Importance of Coherence in Mathematics (4.38 minutes)
- Slide 30: Learning Progressions, Part 2 (3:02 minutes)
- Slide 34: Operations and Algebraic Thinking (1:43 minutes)
- Slide 39: Integer Subtraction and Addition (2:00 minutes)
- Slide 44: Zero Pairs, Manipulatives, and a Real-World Scenario (5:42 minutes)
- Slide 49: Illustrative Mathematics Tasks (3:19 minutes)
- Slide 56: What’s Fun about Surface Area? (7:24 minutes)
- Slide 62: UDL at a glance (4:36 minutes)
Group Facilitator’s Guide: Preparation Notes

Prepare in advance

- Table supplies (e.g., pens, pencils, highlighters, self-stick notes)
- Chart paper (optional)
- Chart markers (optional)
- Computer
- LCD projector
- Speakers for video
- Internet connection

Presentation Options

Group facilitation may include district-wide workshops or site-level professional learning communities. Extra time (or additional sessions) may be built in for grade span or subject matter breakout groups if desired.

- One 7-hour session (large workshops)
- Two sessions (site-level professional learning communities)
  - One 3.5-hour session (Overview, Units 1, and 2)
  - One 3.5-hour session (Units 3 and 4)
Group Facilitator’s Guide: Section Notes

<table>
<thead>
<tr>
<th>Section:</th>
<th>Mathematics: K–8 Learning Progression Overview</th>
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<tbody>
<tr>
<td>Slides:</td>
<td>1–4</td>
</tr>
<tr>
<td>Suggested Time:</td>
<td>10 minutes</td>
</tr>
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</table>

**Intent:** To provide an overview of the module.

**This section includes:**

**Module Goal:** To understand how the Common Core State Standards (CCSS) for Mathematics: Kindergarten through Grade 8 (K–8) Learning Progressions:

- Are sequenced across and within grade-level spans (i.e. Kindergarten through grade 5 [K–5] and grades six through eight [6–8])
- How to apply Mathematics: K–8 Learning Progressions to instructional practice

**Pre-assessment** (5 minutes)

**Outline:**

- **Unit 1: Introducing the Learning Progressions (1 hour)**
  Develop an awareness of the history and rationale of the Mathematics: K–8 Learning Progressions.

- **Unit 2: Unpacking the Learning Progressions (2 hours, 30 minutes)**
  Develop knowledge of the Mathematical Learning Progressions within and across grade level spans.

- **Unit 3: Applying the Learning Progressions (2 hours, 30 minutes)**
  Apply a Mathematics: K–8 Learning Progression to develop the ability to support students' conceptual understanding, technology use in learning, ability to think creatively and reflectively, and/or solve real-world mathematical problems.

- **Unit 4: Summarizing the Learning Progressions (1 hour)**
  Summarize understanding of the Mathematics: K–8 Learning Progressions.

**Presentation Note:**

- Point of Use document:
  - Pre-Assessment
<table>
<thead>
<tr>
<th>Section:</th>
<th>Unit 1: Introducing the Learning Progressions</th>
</tr>
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<tbody>
<tr>
<td>Slides:</td>
<td>5–18</td>
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<tr>
<td>Suggested Time:</td>
<td>1 hour</td>
</tr>
</tbody>
</table>

**Intent:** The participant will develop an awareness of the history and rationale of the Mathematics: K–8 Learning Progressions and how the changes will impact instruction and learning.

**This section includes:**

**Unit 1: Introducing the Learning Progressions**

- History and Rationale (15 minutes)
- Instructional Changes (15 minutes)
- What are the Learning Progressions? (15 minutes)
- Progressions from the Student Perspective (15 minutes)

**Preparation Note:**

- Preview the video for content and to resolve possible technical issues.

**Presentation Note:**

- Point of Use document:
  - Aspects of the Mathematics Standards
- Videos
  - Slide 8: The Mathematics Standards: How They Were Developed and Who Was Involved length (8:11 minutes)
  - Slide 12: The Mathematics Standards and the Shifts They Require (1:14 minutes)
  - Slide 18: The Importance of Mathematical Progressions from the Student Perspective (3:04 minutes)
Group Facilitator’s Guide: Section Notes

<table>
<thead>
<tr>
<th>Section: Unit 2: Unpacking the Learning Progressions</th>
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<tbody>
<tr>
<td>Slides: 19–45</td>
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<tr>
<td>Suggested Time: 2 hours, 30 minutes</td>
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**Intent:** The participant will develop knowledge of the Mathematical Learning Progressions within and across grade-level spans.

This section includes:

**Unit 2: Unpacking the Learning Progressions**

- Definitions, Video and Reflection: K–8 Mathematic Domains (30 minutes)
- Domain Distribution (15 minutes)
- Mathematics Progression focused on CCSS for Mathematics, Standard 6.EE.2 (30 minutes)
- University of Arizona’s Progressions (30 minutes)
- Grade Span Coherence (45 minutes)

**Preparation Note:**
- Preview videos for content and to resolve possible technical issues.

**Presentation Note:**
- Point of Use documents:
  - Slide 21: CCSS for Mathematics excerpt from page 5
  - Slide 31: Learning Progressions, Part 2 Viewing Guide
  - Slide 41: Completed Instructional Alignment Chart for Grades 6–8
  - Slide 42: Instructional Alignment Chart Template
- Videos:
  - Slide 22: Learning Progressions, Part I (8:11 minutes)
  - Slide 28: The Importance of Coherence in Mathematics (4.38 minutes)
  - Slide 30: Learning Progressions, Part 2 (3:02 minutes)
  - Slide 34: Operations and Algebraic Thinking (1:43 minutes)
  - Slide 39: Integer Subtraction and Addition (2:00 minutes)
  - Slide 44: Zero Pairs, Manipulatives, and a Real-World Scenario (5:42 minutes)
Group Facilitator’s Guide: Section Notes

<table>
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<th>Unit 3: Applying the Learning Progressions</th>
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<td>Slides:</td>
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<tr>
<td>Suggested Time:</td>
<td>2 hours, 30 minutes</td>
</tr>
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**Intent:** To gain a deeper understanding of a Mathematics K–8 Learning Progression and instructional strategies that support ALL students.

This section includes:

**Unit 3: Applying the Learning Progressions**

- Illustrative Mathematics Tasks Learning Progression (1 hour)
- “How To” Strategies for English Learners (ELs) (45 minutes)
- Strategies for Students with Disabilities (45 minutes)

**Preparation Note:**
- Preview the video for content and to resolve possible technical issues.

**Presentation Note:**
- Point of Use documents:
  - Slide 50: Academic Language Functions
  - Slide 55: Supports for ELs (Note Taking Guide included on page 2)
  - Slide 56: Evidence of EL Strategies Chart
  - Slide 58: Instructional Strategies for Students with Disabilities
  - Slide 59–61: Universal Designs for Learning Principles

- Handouts (in addition to the Participant Packets):
  - Slides 51–53: Illustrative Mathematics Tasks
    - Grade K.OA.3, Christina’s Candies
    - Grade 1.OA.1, Boys and Girls, Variation 2
    - Grade 2.OA.3, Red and Blue Tiles
    - Grade 3.OA.3, Gifts From Grandma, Variation 1
    - Grade 4.OA.1–3, Comparing Growth, Variation 1
    - Grade 5.OA.2, Video Games Scores
    - Grade 6.EE.2, Distance to School
    - Grade 7.EE.1, Writing Expressions
    - Grade 8.EE.7, The Sign of Solutions

- Videos
  - Slide 49: Illustrative Mathematics Tasks (3:19 minutes)
  - Slide 56: What’s Fun about Surface Area? (7:24 minutes)
  - Slide 62: UDL at a glance (4:36 minutes)
Section: Unit 4: Summarizing the Learning Progressions; Post-assessment; Evaluation

Slides: 64–74

Suggested Time: 1 hour

**Intent:** The participant will summarize and assess his/her understanding of the Mathematics: K–8 Learning Progressions.

**This section includes:**

**Unit 4: Summarizing the Learning Progressions**
- Summary and Take-Aways (30 minutes)

**Post-assessment (10 minutes)**

**Workshop Evaluation (10 minutes—optional)**

**Preparation Note:**
- Preview the video for content and to resolve possible technical issues.

**Presentation Note:**
- Point of Use documents needed:
  - Slide 67: Unit 1 Summary
  - Slide 68: Unit 2 Summary
  - Slide 69: Unit 3 Summary
  - Slide 71: Post-assessment
  - Slide 74: Workshop Evaluation Form (optional; locally provided)
Independent User’s Guide

Navigation Guide

All of the pages for this professional learning module are listed in the Table of Contents at the left of your computer screen. You can access any page through the corresponding link.

In addition, you may use the next and previous buttons at the bottom of any page to navigate to other pages. Refer to the illustration below:

System Requirements

The Brokers of Expertise Web site is optimized for the following Web browsers: Google Chrome, Microsoft® Internet Explorer 8.0 or higher, Mozilla Firefox (ESR recommended), and Safari 5.0 or higher.

You may encounter problems viewing professional development content or resources if you use an earlier version or a Web browser that is not listed above.
Independent User’s Guide

Visit one of the following links to upgrade your browser to a current version:

- Google Chrome - [http://www.google.com/chrome](http://www.google.com/chrome)
- Mozilla Firefox - [Windows | Macintosh](http://www.mozilla.com/firefox/)

### Plug-Ins

Plug-ins are additional software programs that allow your Web browser to perform special functions, like viewing audio or video clips. Some of the materials available through the professional learning modules may require a particular plug-in to access content. You can find links to download and install some frequently-used plug-ins here:

- Adobe Flash Player - [http://get.adobe.com/flashplayer](http://get.adobe.com/flashplayer), Provides access to video clips or interactive Flash resources

### Pop-up Windows

Pop-up windows, or “pop-ups,” are windows that appear when you click on certain links throughout the module. You may not be able to see these windows if you have a “pop-up blocker” or have disabled pop-ups in your Web browser. Please enable pop-up windows in your Web browser preferences so that you can see all content in this module.

### Need Help?

Contact the Brokers of Expertise Help Desk if you have questions or need assistance. You may use the contact form or send help requests to [webmaster@myboe.org](mailto:webmaster@myboe.org).
Independent User’s Guide

Materials and Equipment

Download and print handouts, if desired (can be read online as well):

• Handouts:

  Unit 2, Activity 4:
  
  o University of Arizona’s Progressions, K, Counting and Cardinality; K–5, Operations and Algebraic Thinking
  
  o University of Arizona’s Progressions, 6–8 Expressions and Equations

  Unit 2, Activity 5
  
  o Completed Instructional Alignment Chart 6–8 The Number System, Integers
  
  o Instructional Alignment Chart
  
  o California’s Common Core State Standards for Mathematics

  Unit 3, Activity 1
  
  o Academic Language Functions
  
  o Illustrative Mathematics Tasks (your teaching grade level, and the grade level above and below)
    
    ▪ Grade K - K.OA.3, Christina’s Candies
    ▪ Grade 1 - 1.OA.1, Boys and Girls, Variation 2
    ▪ Grade 2 - 2.OA.3, Red and Blue Tiles
    ▪ Grade 3 - 3.OA.3, Gifts From Grandma, Variation 1
    ▪ Grade 4 - 4.OA.1–3, Comparing Growth, Variation 1
    ▪ Grade 5 - 5.OA.2, Video Games Scores
    ▪ Grade 6 - 6.EE.2, Distance to School
    ▪ Grade 7 - 7.EE.1, Writing Expressions
    ▪ Grade 8 - 8.EE.7, The Sign of Solutions

• Computer/Tablet

• Speakers and/or headphones

• Internet Connection
Math Supports for English Learners


Math Supports for Students with Disabilities


21st Century Skills Resources:


Harding University Math Specialist (blog). 2012.  


Inside Mathematics. 2012. 2nd Grade Content Standards.  

ISTE International Society for Technology in Education. 2012.  
