Teaching Algebra with The Geometer’s Sketchpad®
UCB Extension X305.5– Education (2 semester units)

COURSE DESCRIPTION
This is a professional-level, moderated, online course in the use of The Geometer’s Sketchpad software for teaching mathematics, particularly beginning and advanced algebra. In addition to instruction in how to use the software, the course will offer participants pedagogical guidance on how to implement the use of Sketchpad™ in their classrooms and promote discussion on how dynamic mathematics software affects the teaching and learning of algebra. The course runs for six weeks with a scheduled start and end date and is structured into six weeklong units. While participants have flexibility within each week, the course is synchronous, meaning that participants are expected to begin and complete the activities for each week during the week they are assigned.

COURSE OBJECTIVES
After participants complete this course, they will be comfortable using Sketchpad both as an investigation tool and as a demonstration tool, and will be able to

- Illustrate slope and other graphing concepts in the coordinate plane
- Create dynamic graphs using a variety of techniques
- Explore functional relationships within geometric figures
- Use dynamic environments to explore properties of numbers and operations
- Construct alternative representations of functional relationships that can lead to deeper understanding
- Explore sequences and functions through iteration
- Create demonstrations that involve animation and action buttons
- Appreciate the pedagogical implications of exploring algebra in a dynamic environment

INTENDED AUDIENCE
This course is intended for secondary and middle school mathematics teachers and instructors of pre-service teachers. Although the course content focuses on algebra concepts, any current or prospective teacher can learn how to use Sketchpad to supplement the middle or high school mathematics curriculum.

PREREQUISITES
Participants should be familiar with middle school or high school algebra concepts. They should also be comfortable using computers and must have access to the Internet and The Geometer’s Sketchpad software.
METHODS OF INSTRUCTION

Each week follows the same structure in which participants complete these activities:

- Interact with a dynamic sketch that introduces the week’s mathematical focus
- Watch three videos (each approximately five to eight minutes long): an interview with a Sketchpad developer or teacher educator, an interview with a classroom teacher that uses Sketchpad, and a tutorial of Sketchpad features introduced that week
- Download six to eight PDFs of Sketchpad activities and any associated sketches
- Complete these activities offline using Sketchpad
- Participate in an asynchronous discussion forum (as well as an optional scheduled chat)
- Complete a project using Sketchpad
- Reflect by responding to specific prompts in an online journal

Some weeks include optional activities or videos for enrichment or additional support.

GRADE BREAKDOWN

In order to receive credit for the course, participants must complete all weekly activities, turn in all six projects, respond to all six journal prompts, and participate in the class discussion forum at least twice each week. Assessment is project-based and all six weeks are weighted equally.

The two units offered for this course are based upon the expectation that each week will require about five hours of work (at least two hours for online activities – visual media, discussion forums, and the journal – and at least three hours for offline Sketchpad activities and the project).

The instructor will evaluate and provide feedback for submitted work and class participation. Grades will be assigned on a standard percent scale based on the following breakdown:

<table>
<thead>
<tr>
<th>Activity Type</th>
<th>Percentage</th>
</tr>
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<tbody>
<tr>
<td>Activity checks</td>
<td>20%</td>
</tr>
<tr>
<td>Journal entries</td>
<td>20%</td>
</tr>
<tr>
<td>Discussion forums</td>
<td>20%</td>
</tr>
<tr>
<td>Weekly projects</td>
<td>40%</td>
</tr>
</tbody>
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REQUIRED TEXTS AND MATERIALS

All written material will be Sketchpad activities provided as PDFs, primarily from these books:


Participants are not required to purchase these books or any other materials (other than The Geometer’s Sketchpad software) in order to take this course.
SESSION-BY-SESSION SUMMARY

Week 1: Points and Lines
Using Sketchpad’s freehand tools and measurement capabilities, participants explore points, lines, and slope in the coordinate plane.

Visual Media:
- “A Formula for Slope” JavaSketch
- Interview with Nick Jackiw, “Moving between Shape and Number”
- Interview with Jennifer North Morris, “The Slope Game”
- Video demonstration with Janet Bowers, “Points and Lines”

Activities:
- Points Lining Up in the Plane
- Rate of Change
- The Slope of a Line
- The Slope Game
- The Slope-Intercept Form of a Line
- Solving Systems of Equations
- Slopes of Parallel and Perpendicular Lines

Slope Presentation Project:
Create a demonstration sketch that illustrates slope and the calculation of slope.

Journal Prompt:
Participants describe how to construct lines with given characteristics, and they reflect on how the activities and discussions impacted their thinking about teaching slope.

Week 2: Functional Relationships
Participants learn to create robust constructions of geometric objects and use these constructions to explore functional relationships with geometric figures.

Visual Media:
- “Fixed Perimeter” JavaSketch
- Interview with Nick Jackiw, “Functional Dependency”
- Interview with Rachel Chou, “Graphing for a Purpose”
- Video demonstration with Janet Bowers, “Functional Relationships”

Activities:
- Old-Style Plotting
- Circumference Function
- Functions in a Triangle
- Direct Variation
- Inverse Variation
- Functional Geometry

Functional Geometry Project:
The project is to construct a rectangle with a fixed perimeter and create several representations of the functional relationship between a side length and the area.

Journal Prompt:
Participants choose one of the functional geometry activities and describe the prerequisite knowledge needed to do the activity and the algebraic concepts addressed in the activity.
Week 3: Families of Functions
Participants learn several techniques for constructing dynamic families of functions, and explore quadratic, exponential, and linear functions through dynamic constructions.

Visual Media:
- “Sine Transformations” JavaSketch
- Interview with Nick Jackiw, “Building Models”
- Interview with Nathalie Sinclair, “Teaching Sketchpad to Calculate Slope”
- Video demonstration with Janet Bowers, “Families of Graphs”

Activities:
- Graphing Quadratic Functions
- Exponential Functions
- The Point-Slope Form of a Line
- Exploring Parabolas in Vertex Form
- Quadratic Intercepts
- Translating Functions

Ferris Wheel Project:
The project is to create a sine function whose amplitude and period are controlled by an independent point.

Journal Prompt:
Participants are asked to discuss common student misconceptions about the use of letters in algebra, such as confusing coefficients and variables. Participants are asked to discuss how varying a coefficient (to show a family of functions) is different from varying a variable.

Week 4: Number Properties
Participants use dynamic environments to explore properties of numbers and operations.

Visual Media:
- “Sum and Product” JavaSketch
- Interview with Nathalie Sinclair, “Taking Manipulatives Further”
- Interview with Steve Rasmussen and Nathalie Sinclair, “Raz’s Four Function Machine”
- Video demonstration with Janet Bowers, “Number Properties”

Activities:
- Squares and Square Roots
- Exponents
- Zero and Negative Exponents
- Raz’s Magic Multiplying Machine
- Mystery Machines
- Equivalent Expressions: The Border Problem
- The Product of Two Binomials

Four Operations Machine Project:
The project is to construct a “Machine” in Sketchpad that visually represents the sum, difference, product, and quotient of two values on the number line.

Journal Prompt:
Teachers reflect on how the activities, videos, and discussions have influenced their thinking about how to teach properties of numbers and operations.
**Week 5: Multiple Representations**
Participants explore several dynamic models for solving equations and representing functions.

Visual Media:
- “Dynagraphs” JavaSketch
- Interview with Nick Jackiw, “Objects in Motion”
- Interview with Steve Rasmussen and Nathalie Sinclair, “Mystery Machine”
- Video demonstration with Janet Bowers, “Multiple Representations”

Activities:
- Introducing Dynagraphs
- From Dynagraphs to Cartesian Graphs
- Approximating Solutions to Equations
- Solving Linear Equations by Jumping
- Solving Linear Equations by Balancing
- Trigonometry Tracers

Dynagraph Project:
The project is to construct dynagraphs to represent three or more functions.

Journal Prompt:
Participants are asked to reflect on the benefits and drawbacks of constructing their own sketches or manipulating premade sketches.

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**Week 6: Dynamic Algebra: Iteration and More**
Participants use iteration to explore ratios, functions, and sequences, and see how dynamic algebra can enrich the study of other algebra topics, such as parametric equations and linear programming.

Visual Media:
- “Sequences” JavaSketch
- Interview with Nick Jackiw, “The Intersection of Two Functions”
- Interview with David Hallas, “Variables that Vary”
- Video demonstration with Janet Bowers, “Iteration”

Activities:
- Fractals and Ratios
- Functions Again and Again
- Generating Sequences
- Linear Motion
- Box and Whiskers
- Linear Programming

Final Project:
The final project is to create a dynamic sketch with animation that can be used as a demonstration in an algebra class.

Journal Prompt:
Participants are asked to reflect on what dynamic algebra means to them and how the dynamic nature of Sketchpad changes the way students can learn and understand algebra.