8.EE The Sign of Solutions

Alignment 1: 8.EE.7

Grade	8
Domain	EE: Expressions and Equations
Cluster	Analyze and solve linear equations and pairs of simultaneous linear equations.
Standard	Solve linear equations in one variable.

Without solving them, say whether these equations have a positive solution, a negative solution, a zero solution, or no solution.

a. $3x=5$

b. $5z+7=3$

c. $7-5w=3$

d. $4a=9a$

e. $y=y+1$

Commentary:

It is possible to say a lot about the solution to an equation without actually solving it, just by looking at the structure and operations that make up the equation. This exercise turns the focus away from the familiar “finding the solution” problem to thinking about what it really means for a number to be a solution of an equation. Work that students do in high school related to the domain A-SSE Seeing Structure in Expressions will naturally build on the sensibilities developed with this type of task.

It would be a good idea for teachers model, in a question-and-answer format, the reasoning process expected with one or two examples. Otherwise, they might just base their answers on mental transformations of the equations and not on the structure of the equations.

Adapted from Algebra: Form and Function, McCallum, Wiley, 2010.

Solution: Reasoning About Operations

a. $3x=5$ has a positive solution. Since 3 is positive, the second factor in the product $3x$ also has to be positive to produce the positive result 5.

b. $5z+7=3$ has a negative solution. This is because we are adding 7 to the number $5z$, and get a result of 3. The only way for that to happen is to have started with a negative number. So $5z$ must be negative, and since 5 is positive, that is only true when z is negative.

c. $7-5w=3$ has a positive solution. We know this because we are starting with 7, a positive number, and we are ending at 3, which is a smaller number. To get there we are subtracting $5w$, which must also be positive in order for us to get from 7 to 3. Since $5w$ is positive, it follows that w must be positive.

d. $4a=9a$ has zero as its solution. 4 multiplied by a number can never equal 9 multiplied by the same number, unless that number is zero.

e. $y=y+1$ has no solution. If it did, we would be saying that some number, y, is equal to the number that is one more than itself, which is impossible.

Solution: Solutions that are intended to be understandable to kids.

a. If $3x=5$, then x is positive. Five is the product of 3 and x. 5 is positive. For a product to be positive, both factors must have the same sign. 3 is positive, so x must be positive.

b. If $5z+7=3$, then z is negative. We are adding something to 7 and getting something smaller than 7. What we are adding is $5z$. This must be negative, so z must be negative.

c. If $7-5w=3$, then w must be positive. We are subtracting $5w$ from 7 and getting something smaller than 7. So what we are subtracting must be positive. If $5w$ is positive, then w is positive.

d. If $4a=9a$ then $a=0$. If $a=1$, then the equation is saying $4=9$, so 1 is not a solution. If $a=2$, then the equation is saying $8=18$, so 2 is not a solution. If $a=1/2$, then the equation is saying $2=4.5$, so $1/2$ is not a solution. If $a=-3$, then the equation is saying $-12=-27$, so -3 is not a solution. Four things can never be equal to 9 things, since there are 5 more things in 9 things than in 4 things. But the exception is when the things are nothings. If you have 4 nothings and I have 9 nothings, then we both have nothing. Plenty of nothing is nothing. A bigger plenty of nothing is still nothing.

e. There is no number that stays the same when we add 1 to it. So there is no number y that makes $y=y+1$ true.